Physics Lecture 4 - Do Smaller Axles Really Go Faster ?

Summary/Conclusion

Theoretical reasoning and experimental techniques have been applied in the Jobe Consulting labs to show that the size of an axle in a graphite lubricated journal bearing axle/wheel system does *not* affect frictional drag on the rolling wheel system. The popular misconception that small axles go faster can be blamed on an extrapolation of 3rd class lever results from large to small axles as shown on this page. On the following page we apply the conservation law of work/energy for the correct result and back it up with data.

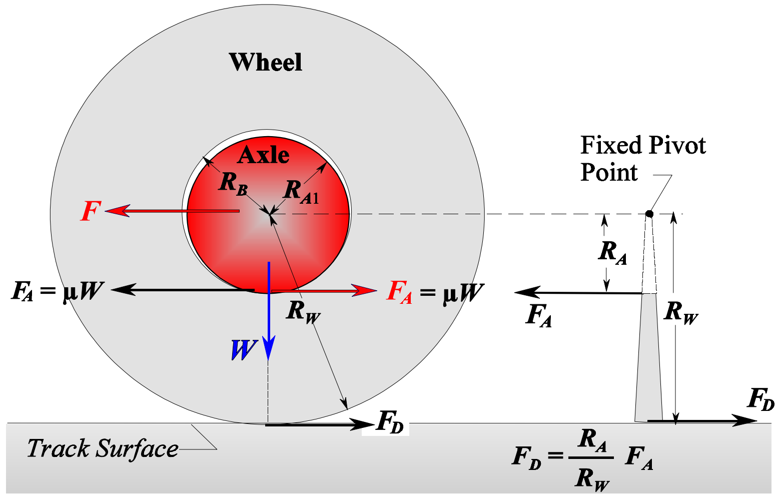


Figure 1. *Forces on a wheel and axle.*

Theoretical Analysis

Consider a wheel/axle system as shown in **Fig. 1**. The axle and horizontal forces on it are shown in **red**, and the wheel is gray with its horizontal forces shown in **black**. Definitions are as follows:

F is an applied force on the axle perhaps caused by gravity on the body in which the axle is imbedded.

F_A is a force to the right on the bottom of the axle caused by dragging it along the surface of the wheel bore. **Lecture 2** shows this force = μW

W is the weight or load (e.g. the body) supported by the wheel bore/axle interface which has coefficient of friction μ.

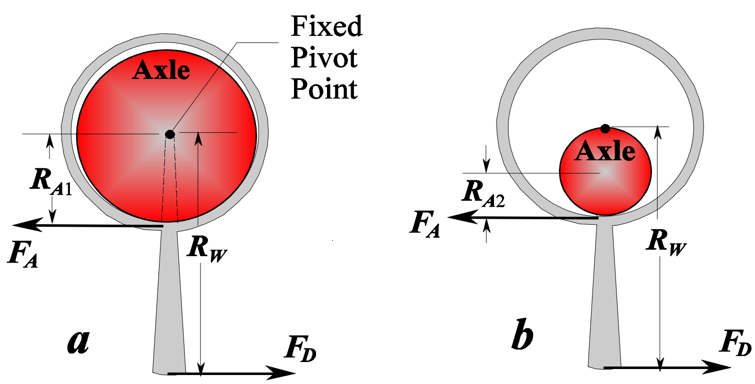


Figure 2. *The wheel as a type 3 lever.Only wheel forces shown.*

F_A is a force to the left on the surface of the wheel bore, equal and opposite to F_A, caused by the axle drag.

F_D is the resistance of the system to the applied force F, and under constant velocity is equal and opposite to F, and pulls to the right on the wheel bottom at the track surface contact point.

The large axle of radius R_{A1} is just slightly smaller than the bore radius R_B . The wheel radius is R_W. If you could take a slice of the wheel, as shown to the right in **Fig. 1**, and consider it relative to the body axle/wheel system, it would behave like a 3rd class lever relative to a person sitting on the car. Such a person would see the lever with an effective fixed pivot point close to the center of the axle. To see this consider a section of the rigid wheel cut out like a tennis racket (less strings) as shown in **Fig. 2*a***. The section of wheel shown rotates around the axle with very little clearance, making in effect a fixed pivot point near the bore center. The equation relating forces and distances for a rigid body acting as a type 3 lever is

$$F_D = \frac{R_{A1}}{R_W} F_A \qquad (1)$$

This equation seems to make sense because we know for example that brake drum friction is more effective in stopping a car if the drum diameter is fairly large compared with the wheel diameter. But what if the axle were much smaller as in **Fig. 2 *b*** ? We know that the force $F_A = \mu W$ will be the same because **Lecture 2** showed as long as weight didn't change a different apparent contact area would not make any difference. So then according to Eq. (1) if we make the axle smaller such as R_{A2} in **Fig. 2 *b*** then the drag force F_D must be smaller. *Wrong!* We can't assume this. Now we don't have a fixed pivot point because there is so much play between the axle and wheel bore. So we really can't say we still have a type 3 lever. Eq. (1) turns out to be OK as long as the axle has about the same diameter as the bore, but for the axle significantly smaller we need a different approach as follows.

The best way to analyze this problem is to use the conservation of work/energy. Consider a huge wheel bore like a hula hoop and a small cylindrical axle at point **P** carrying a weight W. Now the bore radius is just slightly smaller than the wheel radius as seen in **Fig. 3 *a***. In **Fig. 3 *b*** we again apply force $\boldsymbol{F}$ to move the axle to the left a horizontal distance ℓ as the axle slides on the inside bore surface. Point **P** thus rotates through angle φ. So the frictional work done is the sliding force times the distance it slides. This distance is the equivalent arc length on the bore surface given by $\ell = R_B\,\varphi$ where φ is the angle of rotation. So the work done by the axle force against friction is $F_A\,R_B\,\varphi$. The work done to move the whole system, including the wheel, to the left against the drag force $\boldsymbol{F_D}$ at the track surface is $\boldsymbol{F_D}$ times the distance it is applied, which is $\ell = R_W\,\varphi$. So the whole system drag work is $F_D\,R_W\,\varphi$. The work done to move the whole system must equal the frictional energy dissipated, so that

$$F_D\,R_W\,\varphi \;=\; F_A\,R_B\,\varphi \tag{2}$$

$$F_D \;=\; F_A\,\frac{R_B}{R_W} \tag{3}$$

$$F_D \;=\; \mu\,W\,\frac{R_B}{R_W} \tag{4}$$

Thus we have the interesting result that the frictional drag on a car using dry lubed journal bearings is independent of the axle size but rather depends only on bore size. Of course when we have the axle size a close fit to the bore size then Eq (1) is pretty accurate *but for that case only*. Notice also that when we let R_B and R_W approach ∞ then the wheel becomes just a flat strip and we have the frictional drag $F_D = \mu W$ on a level surface just like in Lecture 2.

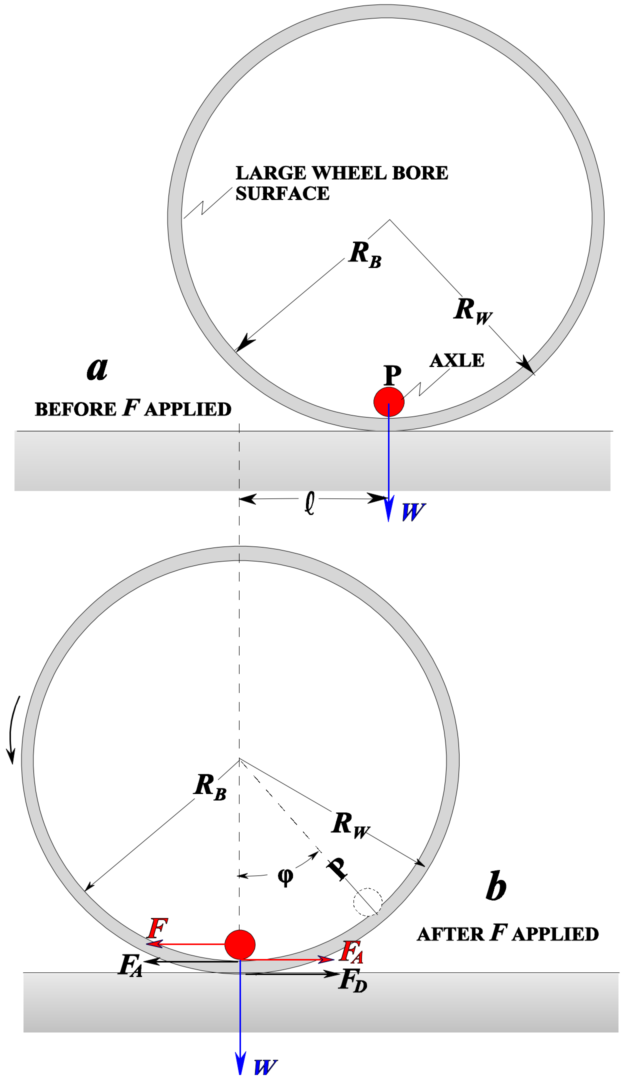


Figure 3. *Showing a small wheel axle sliding inside a large wheel bore.*

Experimental Procedure

We pick the John Silver Narrow (JNSN) car described in the ***Physics of the Pinewood Derby*** book for the tests. As shown in **Figure 4** the car has 92% of its weight on and including the 2 rear wheels (128.85 g) and 8% on and including the left front wheel (11.57 g). The right front wheel does not touch the track and is counted as part of the body weight. So our tests will only need to use the rear axle/wheel pair.

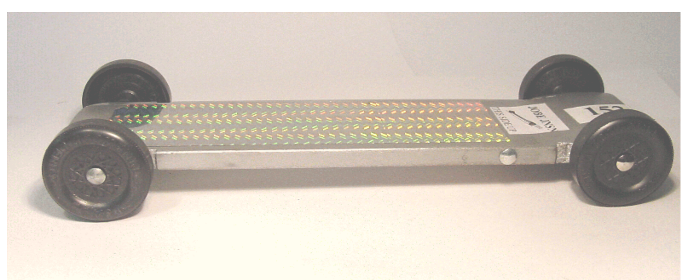


Figure 4. *The JNSN car is used in these tests.*

Figure 5 shows the 3 axle sizes tested. The 0.086" axle is a Hodges (see Lecture 3) nickel plated turned down to 0.062" on the end to fit the rear of the JNSN car. This car routinely runs the 0.062" axle (center) and has a 0.065" body hole. The 0.062" axle is made from a polished hard tool steel drill bit shaft inserted into a standard sized head. The top axle is a 0.045" dia. sold as a plated hardened steel hinge pin for model airplane ailerons. **Fig. 6** shows the wheel bores. All are lubed with Super-Z, the JNSN 4 times.

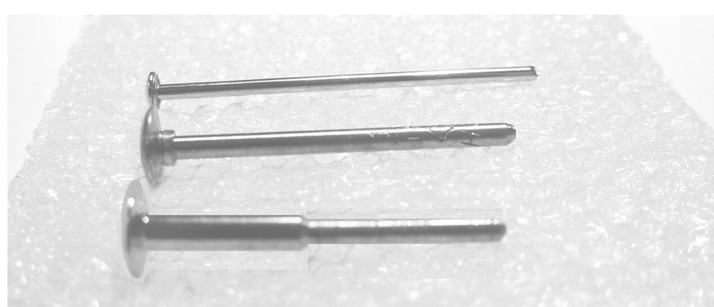


Figure 5. *Axles: Btm = 0.086", Center = 0.062", Top = 0.045"*

Experimental Data

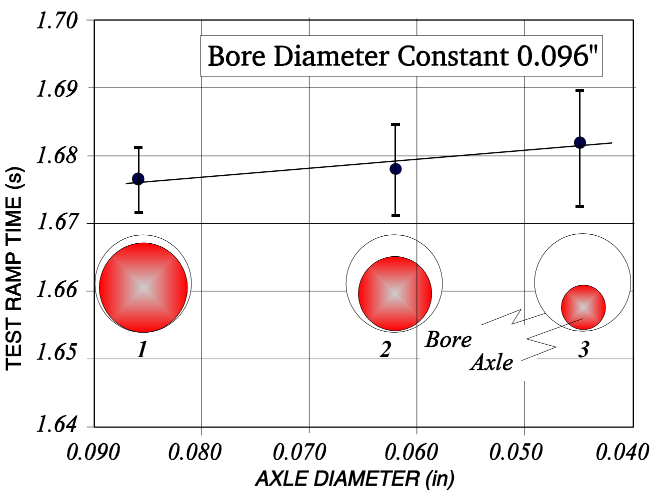
The JNSN car was run on the Friction Test Ramp (FTR) as shown in **Lecture 3** (ramp travel = 29"). Four runs were made of different axle/wheel bore diameter combinations. **Table 1** shows the complete data of Test 1 consisting of 5 runs on lane A and 5 runs on lane B. The Blue Streak was run as a control car (it shows slight slowing because it was lubed only once at the start). AVG denotes averages of each 5 run heat and STD is the standard deviation of the data. It means that a 6th run would have a 2 in 3 chance of being within ±1 STD and 9 in 10 chance of being within ±2 STD. OAVG and OSTD refer to overall values for all 10 runs of the entire Test. **Table 2** shows the overall results of the 4 Tests and **Figure 7** shows a graphical display of the results. As axle diameter decreases with the same bore size, the times remain the same to within experimental error. There is even a tendency for times to increase slightly. On the other hand, keeping the axle diameter constant, as in the bottom of **Figure 6**, we see a sharp decrease in run time as the bore size is decreased.

Fig. 6. *Bores: Right standard. Left reduced using epoxy resin.*

In Tests 1- 3 a wheel rotates 7.76 times as it travels 29" down the test ramp and the axle rubs on the 0.096" bore surface for a linear rub distance of 2.35" = (7.76)(0.096π). In Test 4 the axle rubs on the 0.066" bore surface for only 1.62". So proportionally less frictional energy is dissipated . QED.

Table 1 - Times for test 1- JNSN Bore = 0.096" Axle = 0.086"

BLUE STREAK		JNSN	
0.105" Bore/0.089" Axles		0.096" Bore/0.086" Axles	
Ramp	Time	Ramp	Time
B	1.6648	A	1.6601
B	1.6530	A	1.6757
B	1.6648	A	1.6855
B	1.6702	A	1.6851
B	1.6511	A	1.6833
AVG	1.6608		1.6779
STD	0.0074		0.0096
A	1.6613	B	1.6708
A	1.6701	B	1.6766
A	1.6728	B	1.6754
A	1.6685	B	1.6787
A	1.6667	B	1.6731
AVG	1.6679		1.6749
STD	0.0039		0.0027
OAVG	1.6643		**1.6764**
OSTD	0.0056		**0.0062**



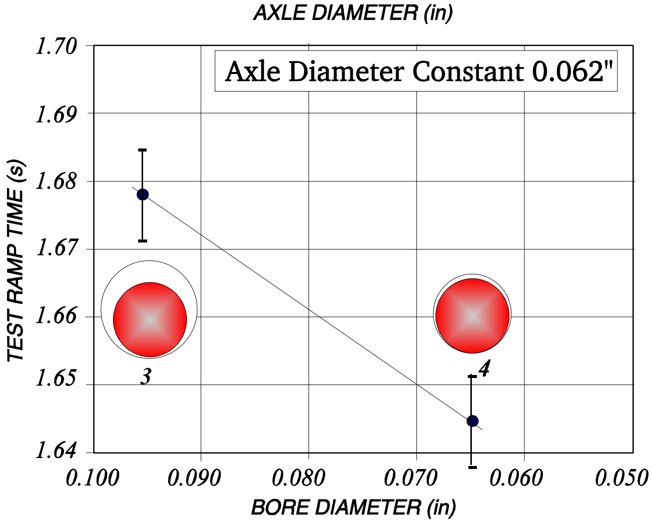


Figure 7. *The results of Table 2 data in graphical form.*

Table 2 - Time overall average results for all 4 tests of JNSN

Stats	BLUE STREAK	JNSN
	Times	Times
Test 1	All 0.105" Bore/0.089" Axles	0.96" Bore/0.086" Axles
OAVG	1.6643	**1.6764**
OSTD	0.0056	**0.0062**
Test 2		0.96" Bore/0.062" Axles
OAVG	1.6796	**1.6784**
OSTD	0.0076	**0.0076**
Test 3		0.96" Bore/0.045" Axles
OAVG	1.6858	**1.6820**
OSTD	0.0100	**0.0148**
Test 4		0.66" Bore/0.062" Axles
OAVG	1.6820	**1.6435**
OSTD	0.0097	**0.0100**