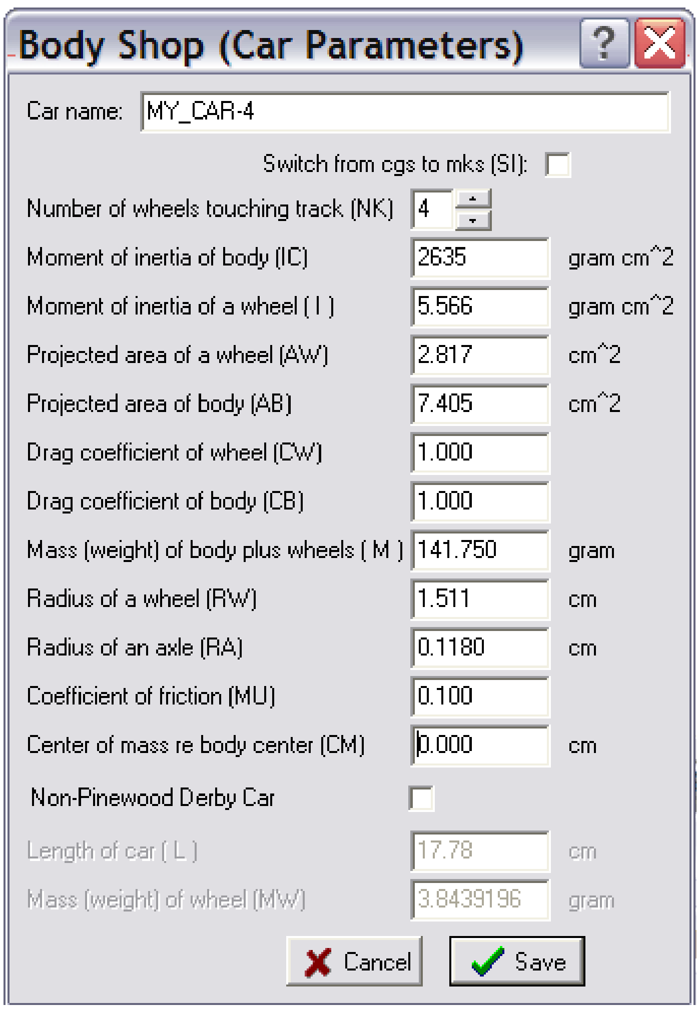
Physics Lecture 25 - Are 3 Wheels Rolling Really Faster than 4 Rolling?

Introduction

The short answer is "sometimes". Again, we have here an opportunity for the VR-II to teach some fundamental Pinewood Derby physics. In the previous Lecture 24, we saw the effects of center of mass placement on both the inclined plane ramp track and the curved or sagging circular arc ramp track. And, we also saw how even a slight amount of air resistance could have an impact on which car had the best finish time. In this lecture, we will show some previously unappreciated detail on how the wheel moment of inertia can impact finish times. For identical wheels, when 4 are rolling you will have a factor of 4/3 or 33% more wheel rotational energy than with just 3 wheels rolling. Remember, moment of inertia is to a rotating body what inertial mass is to a body traveling in a translational (zero rotation) trajectory. More on this later. First, lets set up a quick virtual race.

Virtual Race Setup

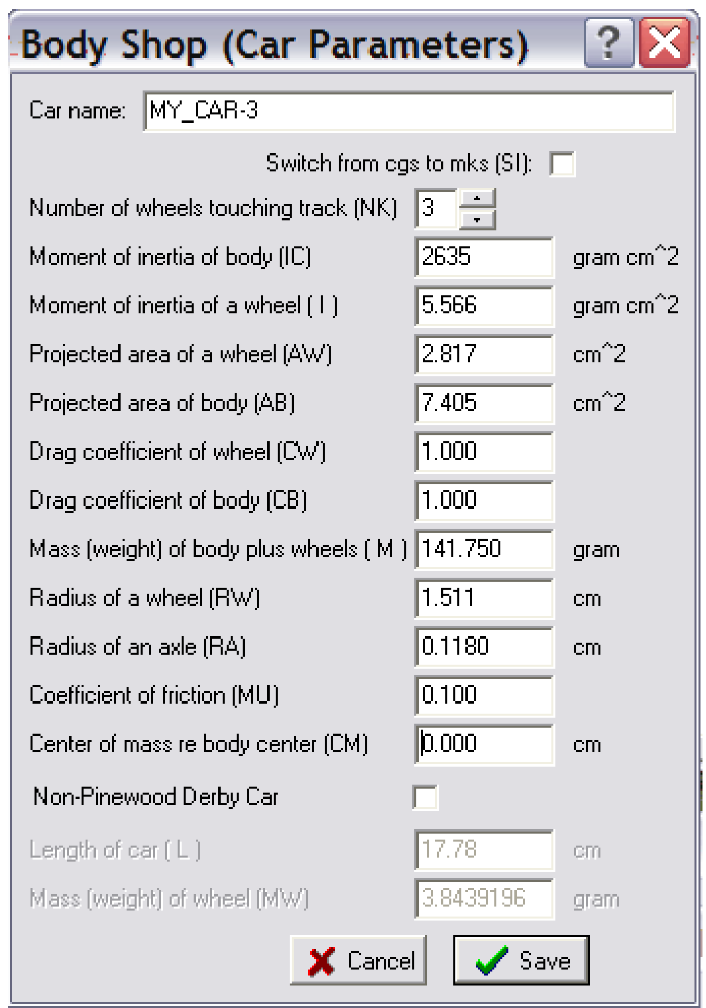
The virtual car MY_CAR comes with the VR-II as the editable car with parameters already specified. Go to the Car drop down menu, select this car, and click [Edit Car Parameters]. All you have to do is change the name to MY_CAR-4 and save as in **Fig 1.** Then change number of wheels touching the track (NK) to "3" and rename the car MY_CAR-3 and save as in **Fig2.** So now we have two identical cars except one has a raised front wheel that does not roll.


Body Shop (Car Parameters)

Car name:	MY_CAR-4	
Switch from cgs to mks (SI):		
Number of wheels touching track (NK)	4	
Moment of inertia of body (IC)	2635	gram cm^2
Moment of inertia of a wheel (I)	5.566	gram cm^2
Projected area of a wheel (AW)	2.817	cm^2
Projected area of body (AB)	7.405	cm^2
Drag coefficient of wheel (CW)	1.000	
Drag coefficient of body (CB)	1.000	
Mass (weight) of body plus wheels (M)	141.750	gram
Radius of a wheel (RW)	1.511	cm
Radius of an axle (RA)	0.1180	cm
Coefficient of friction (MU)	0.100	
Center of mass re body center (CM)	0.000	cm
Non-Pinewood Derby Car		
Length of car (L)	17.78	cm
Mass (weight) of wheel (MW)	3.8439196	gram

Cancel Save

Fig 1 - MY_CAR *parameters that come with the VR. Rename as MY_CAR-4 and save.*


Body Shop (Car Parameters)

Car name:	MY_CAR-3	
Switch from cgs to mks (SI):		
Number of wheels touching track (NK)	3	
Moment of inertia of body (IC)	2635	gram cm^2
Moment of inertia of a wheel (I)	5.566	gram cm^2
Projected area of a wheel (AW)	2.817	cm^2
Projected area of body (AB)	7.405	cm^2
Drag coefficient of wheel (CW)	1.000	
Drag coefficient of body (CB)	1.000	
Mass (weight) of body plus wheels (M)	141.750	gram
Radius of a wheel (RW)	1.511	cm
Radius of an axle (RA)	0.1180	cm
Coefficient of friction (MU)	0.100	
Center of mass re body center (CM)	0.000	cm
Non-Pinewood Derby Car		
Length of car (L)	17.78	cm
Mass (weight) of wheel (MW)	3.8439196	gram

Cancel Save

Fig 2 - MY_CAR-3 *is created with NK = 3.*

Next, select the editable track MY_TRACK_MY_CITY_C_BT. This track has a curved ramp like the Micro-Wizard track. Then edit this track by making a really long coasting run of 146 ft which is 4450 cm. Also rename the track by changing the old name to anything different before saving, here as track MY_TRACK_MY_CITY_C_BT_4450.

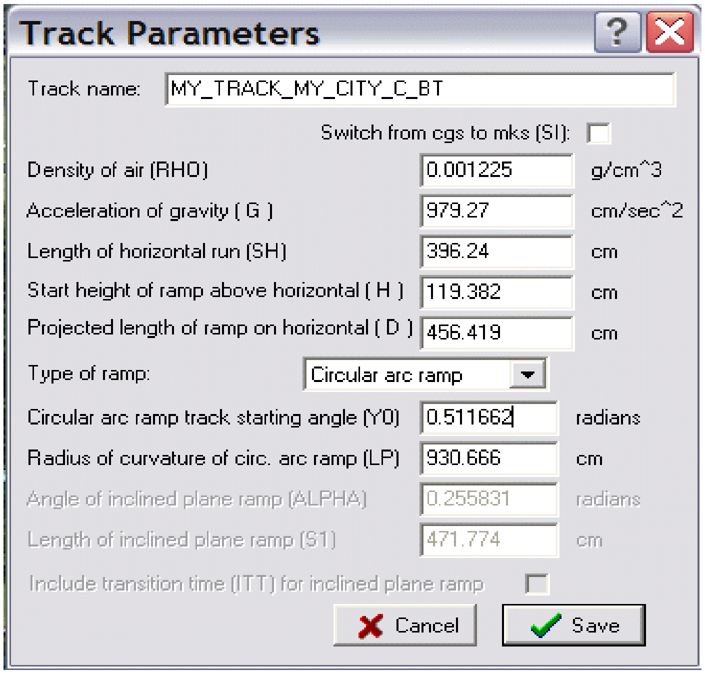


Fig 3 - *Track with editable parameters that comes with the VR.*

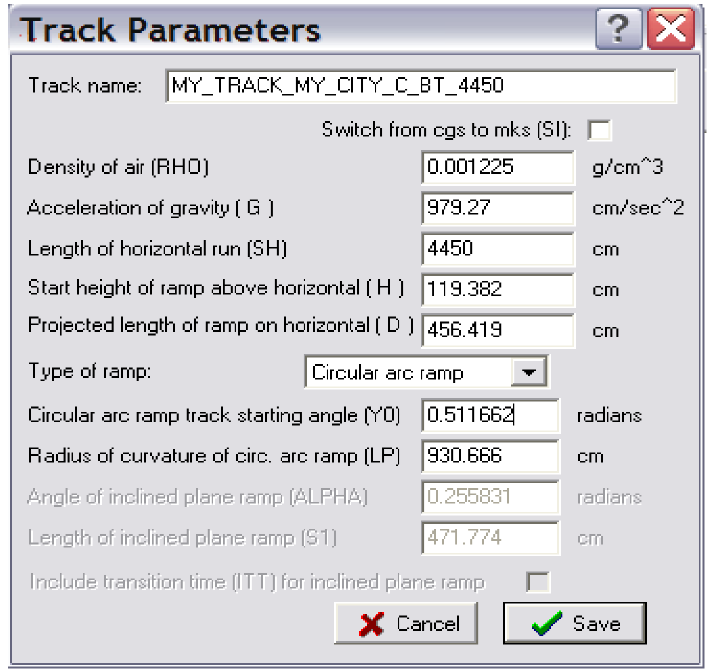


Fig 4 - *Make a really long coasting run of 146 ft = 4450 cm. Then rename track and save it.*

Note on the VR-II run below, the total ramp + coast run = 1.00. Here, the coast is much longer than the ramp, with ratio 4450/456.419, so the coast is about 90% of this unitary distance and the ramp only about 10 %. The cars remain the same size, but their horizontal size relative to the track is correct only when the coast is the usual 13 ft = 396.24cm. So instead of having a tiny car size, we just allow the front bumper to track the correct motion in real time. Click on video.

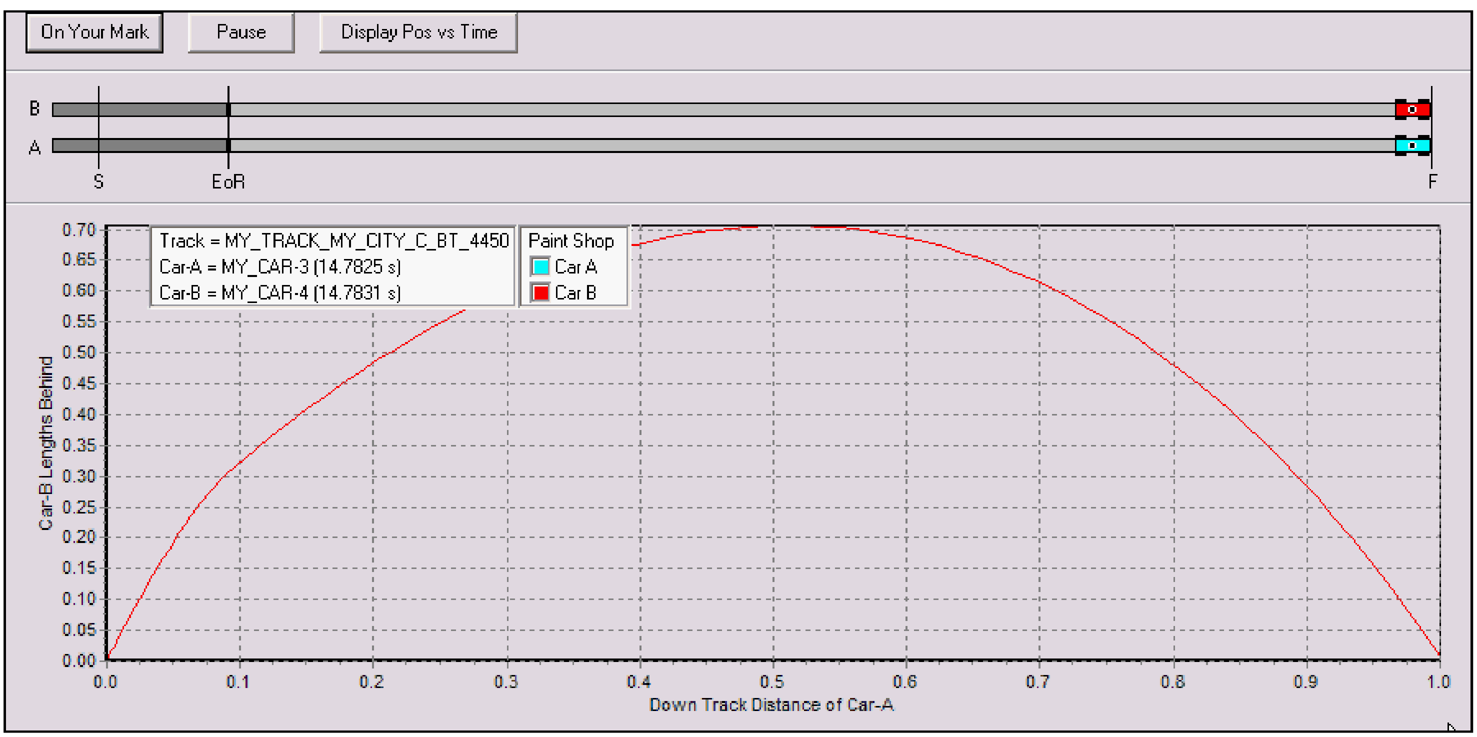


Fig 5. *Video of the Race between the N = 4 and N = 3 wheeled cars. The cars appear jumpy because of the video storage size limit and will run much smoother on the actual VR-II program. Note the car separation red curve.*

Analysis of Motion

Here we analyze the motion to see why the 4-wheel rolling car overtakes the 3-wheel rolling car on a long coasting run. The analysis is stepwise and straightforward and reading it takes less time than doing your income tax. The units will be left out to simplify the analysis. The units for all parameters are those given in **Figs 1** through **4**. To follow the discussion, understand what each step shows before moving on to the next step. Basically, what we are doing first is naming all the forces acting on a car body plus wheels and also naming all its dimensions. Once we have all forces defined, we shall then use Newton's second law to bring them together. We can then compare the deceleration of a 4- wheel car to a 3-wheel car. You may skip the math detail if you wish and go directly to the **Discussion of Results**.

Wheel Specifics

We first will look at a single wheel as in **Fig 6**. We have a wheel radius called R_W with a bore hole radius essentially that of an axle radius, R_A. The wheel is of mass m and rolling as driven by the axle with a velocity $\boldsymbol{v}$. There is a weight W from a fraction of the body mass that presses down through the axle onto the bottom surface of the wheel bore hole. This weight, plus the weight of the wheel, are then supported by the track surface at point P. We will now make a very important point, namely that a wheel that rolls without sliding rotates in direct proportion to the distance traveled. Thus, as shown in **Fig 6**, if the wheel contact point is rolled from Q to P through an angle θ, the curved arc distance is defined as $R_W\,\theta$ provided the angle θ is measured in radians (where 180° = 3.1416 (π) radians). Since there is no slipping, this arc distance is precisely the forward distance s, so that we can write

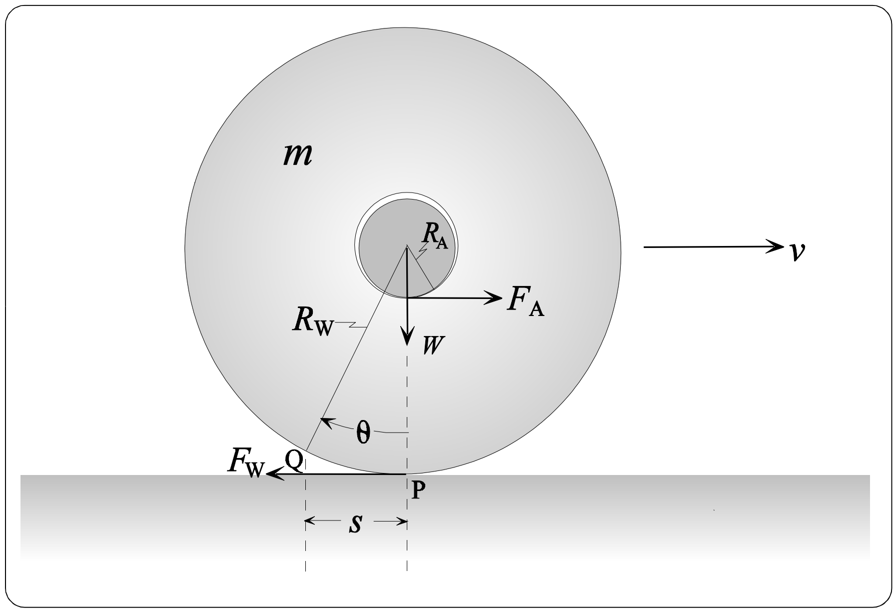


Fig 6 - *Forces and dimensions associated with a wheel.*

$$s = R_W\,\theta \tag{1}$$

System Forces

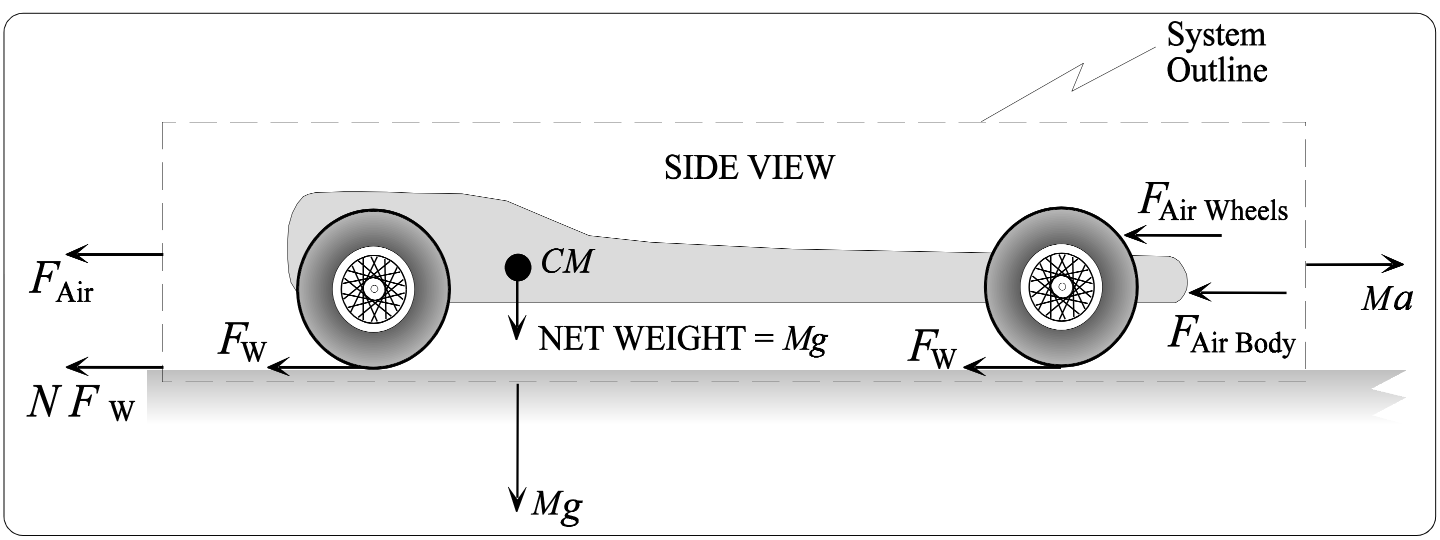


Fig 7 - *Forces associated with entire body plus wheels system.*

Fig 7 shows the forces on the entire moving system, comprised of both body and wheels. There are decelerating (negative) forces composed of air drag and wheel/axle friction. The air drag is the sum of air drag on the body and air drag on all 4 wheels. The force F_{Air} represents the net air drag as one force acting on the entire moving system. And the number of wheels touching, N, times the force F_W on such a wheel, gives the net friction drag against the track.

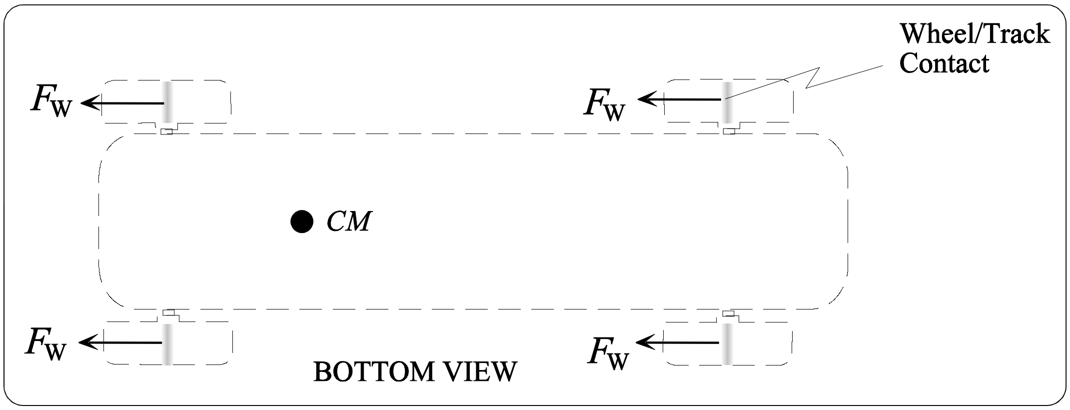


Fig 8 - *Forces where wheels tend to drag against track because of axle/bore friction.*

The wheel/axle friction force was shown in more detail earlier in **Fig 6**, where the rotating wheel bore surface rubbing against the underside of the axle causes a tangential internal force against the bore surface shown as F_A. This force is countered by a backward force F_W at the contact point between the wheel and track surface. In **Fig 8** we present a view of the underside of a "glass" track where we see the 'footprints' where the wheel tread touches the track surface. The drag force F_W on each wheel is shown. Next, we need to set up Newton's second law of motion relative to the forces on the moving system as given in **Fig 7**.

System Acceleration

Newton's second law states that the total moving mass M (body plus wheels = 141.75 grams = 5 oz) times its acceleration a is equal to the total forces acting on M. Thus, from **Fig 7**,

$$Ma = -NF_W - F_{Air} \quad . \tag{2}$$

Note the drag forces act to the left in the negative direction so the acceleration is actually negative, sometimes called "deceleration." Now this equation accounts for all the mass, body and wheels, moving in translational (non-rotational) motion. But rotational motion must also be accounted for, so we must apply Newton's 2nd law to this type motion as well. This law would then read " a rotating mass with a moment of inertia I times its angular acceleration α is equal to the total torque acting on this mass. Recall that torque is a force applied perpendicular to a radius a certain distance from a rotation spin axis and is equal to the force times the application radius. Thus, the wheel of **Fig 6** according to Newton's 2nd law would read as an equation

$$I\alpha = R_W F_W - R_A F_A \quad . \tag{3}$$

Here, I is the moment of inertia of the wheel and α is the angular acceleration of the wheel. It is important to note that the angular acceleration of the wheel is proportional the translational acceleration of the whole car. Recall equation (1) which stated $R_w \theta = s$. Thus the wheel radius R_W times the rate of change of the angle θ must equal the rate of change of the track distance s. Another was to say this is the wheel radius times its angular spin velocity must equal the velocity down the track. Also, R_W times the rate of change of the angular velocity, which is the angular acceleration α, must equal the rate of change of the velocity down the track, which is the acceleration a of Eq (2). Therefore

$$R_w \alpha = a \quad \text{giving} \quad \alpha = \frac{a}{R_w} \tag{4}$$

Thus, substituting the above value for α, Eq (3) can be written

$$\frac{I}{R_W} a = R_W F_W - R_A F_A \quad . \tag{5}$$

Solving Eq(5) for the frictional drag F_W on each wheel, we have

$$F_W = \frac{1}{R_W}\left(I\frac{a}{R_W} + R_A F_A \right) \quad . \tag{6}$$

Putting this value for F_W into the system acceleration Eq (2) gives

$$Ma = -\frac{N}{R_W}\left(\frac{I}{R_W}a + R_A F_A \right) - F_{Air} \quad . \tag{7}$$

Next, rearrange terms in Eq (7) to get the acceleration a on the left,

$$\left(M + N\frac{I}{R_W{}^2} \right) a = -N\frac{R_A}{R_W}F_A - F_{Air} \quad . \tag{8}$$

Then, divide both sides of Eq (8) by the parenthetical coefficient of a to get a in terms of measurable quantities

$$a = -\frac{N\dfrac{R_A}{R_W}F_A}{\left(M + N\dfrac{I}{R_W{}^2} \right)} - \frac{F_{Air}}{\left(M + N\dfrac{I}{R_W{}^2} \right)} \quad . \tag{9}$$

Now the sliding friction force F_A of the axle against the inside bottom surface of the wheel bore is the weight W (see **Fig 6**) pressing the axle down times the coefficient of sliding friction μ. The N wheels touching, each of mass m, are self-supporting, so the net mass supported by N axles is (M - $N m$). Multiple by g to convert this mass to a weight force. Then suppose for the time being that each of N wheels touching supports $\frac{1}{N}$ of the total body weight (M - $N m$)g pressing down. Then on each rotating wheel bottom bore surface we would have a frictional drag force

$$F_A = \frac{\mu}{N}(M - Nm)g \ , \tag{10}$$

so that Eq (9) becomes

$$a = -\frac{\mu g\dfrac{R_A}{R_W}(M - Nm)}{\left(M + N\dfrac{I}{R_W{}^2} \right)} - \frac{F_{Air}}{\left(M + N\dfrac{I}{R_W{}^2} \right)} \quad . \tag{11}$$

The N =4 Case

First we will consider the N = 4 case, where all 4 wheels touch the track and rotate (N = 4, called NK in **Fig 1** & **2**), in which case the body mass supported by axles is the total mass M less the self-supporting wheels, each of mass m. So the total weight force W as shown in **Fig 6** is W = (M - 4m)g. Next, insert the actual numbers from the **Fig 1** body shop edit box to obtain the acceleration, called a_4, for the N = 4 case

$$a_4 = -\frac{0.1\frac{0.118}{1.511}[141.75 - 4(3.84)979.27]}{\left(141.75 + 4\frac{5.566}{1.511^2}\right)} - \frac{F_{Air}}{\left(141.75 + 4\frac{5.566}{1.511^2}\right)} . \tag{12}$$

$$a_4 = -\frac{7.6475(126.39)}{151.506} - \frac{F_{Air}}{151.506} . \tag{13}$$

$$a_4 = -6.3799 - \frac{F_{Air}}{151.506} . \tag{14}$$

We now calculate the air force F_{Air} at the start of the coast where the VR tells us the velocity v_2 = 449.18, so that

$$F_{Air} = \tfrac{1}{2}C_D A_P \rho v_2^2 = (0.5)(1.000)(18.673)(0.001225)(449.18)^2 = 2307.62 \tag{15}$$

The values of C_D and A_P above come from **Fig 1,** where A_P = Area of Body + 4 x Area of one wheel. Also the air density ρ comes from the Track Parameters Edit box. The net deceleration (negative acceleration) becomes

$$a_4 = -6.3799 - \frac{2307.62}{151.5056} = -6.3799 - 15.232 = -21.620 . \tag{16}$$

The N = 3 Case

Next, as shown in **Fig 9**, consider only 3 wheels touch the track and rotate (N = 3). If the right front wheel is raised, the other 3 wheels will support the entire system provided the center of mass lies withing the triangle with apexes lying at the wheel/track contact points.
In this case the body mass supported is the total mass M less the 3 self supporting wheels, each of mas m. So the total weight force pressing down on the 3 rotating wheel axles is W = (M - 3m)g. Suppose that each wheel supports 1/3 of this total weight, then on each wheel

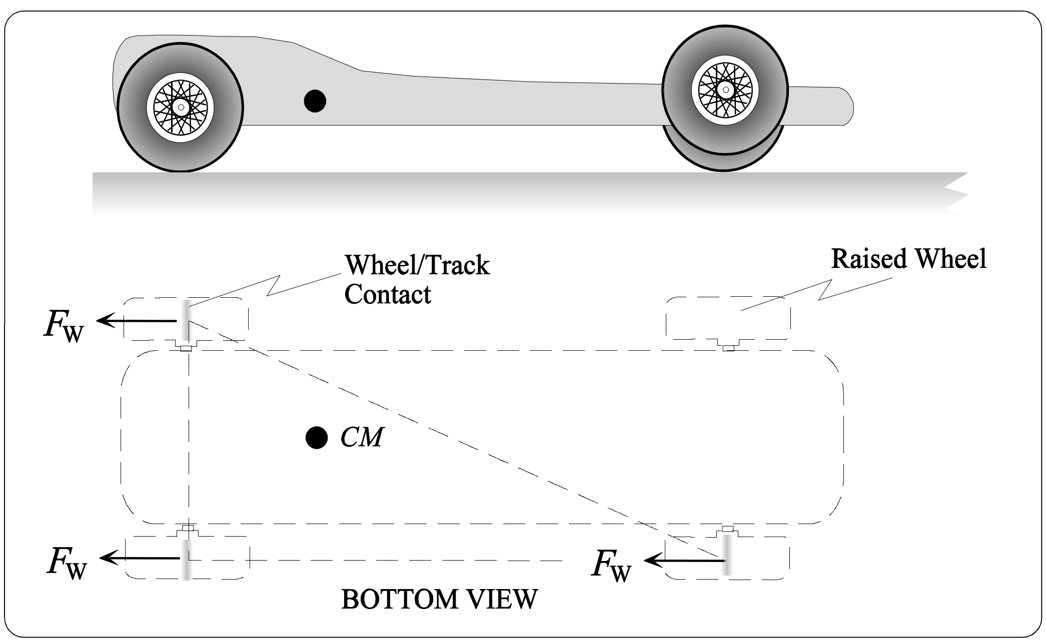


Fig 9 - *The case where one wheel is raised and only 3 wheels roll.*

$F_A = \frac{\mu}{3}(M - 4m)g$. When these values are put into Eq (11) we have the following equation for the net coasting acceleration a_3,

$$a_3 = -\frac{0.1\dfrac{0.118}{1.511}[141.75 - 3(3.84)979.27]}{\left(141.75 + 3\dfrac{5.566}{1.511^2}\right)} - \frac{F_{Air}}{\left(141.75 + 3\dfrac{5.566}{1.511^2}\right)} \ . \tag{17}$$

$$a_3 = -\frac{7.6475(130.23)}{149.064} - \frac{F_{Air}}{149.064} \ . \tag{18}$$

Again, getting the velocity v_2 of the $N = 3$ car from the VR-II [Run] screen, we have the air force given by

$$F_{Air} = \tfrac{1}{2}C_D A_P \rho v_2^2 = (0.5)(1.000)(18.673)(0.001225)(452.46)^2 = 2341.40 \tag{19}$$

$$a_3 = -6.6813 - \frac{2341.40}{149.064} = -6.6813 - 15.708 = -22.389 \ . \tag{20}$$

Discussion of the Results

Table 1 summarizes the results. Both initial air drag and frictional drag are larger for the $N = 3$ car. The larger air drag is totally a consequence of the higher velocity of the $N = 3$ car at the start of the long coasting run because projected area A_P doesn't change. But air drag falls off as the square of the velocity, and the air drag force becomes smaller fairly quickly.

Table 1 - The initial coast deceleration of the $N = 4$ and $N = 3$ Cars

	Friction Drag	Initial Air Drag	Net Initial Deceleration
$N = 4$ Car	6.3799	15.232	21.620
$N = 3$ Car	6.6813	15.708	22.389
Difference	0.3014	0.476	0.769
% Difference	4.8	3.1	3.6

The wheel/axle frictional drag is constant throughout the coast and the $N = 3$ car shows 4.8% more deceleration than the $N = 4$ car. The bottom line is that the $N = 4$ car, although falling behind during the ramp acceleration, immediately at coast start begins a velocity increase relative to the $N = 3$ car. On a short standard horizontal run of 16 to 32 feet, only a small amount of the velocity difference is made up, and $N = 3$ wins. But at about half way on a 146 ft coast run, the $N = 4$ car begins to gain on the $N = 3$ car, and at 146 ft (4450 cm) the $N = 4$ will pass $N = 3$.

The largest effect is the frictional change. Note that when one front wheel is raised, it increases the net mass and weight that must be carried by the other 3 axles. See in Eq (13) the 126.39 for $N = 4$ where in Eq (18) it is 130.23 for $N = 3$. This amounts to 3% of the 4.8 % in **Table 1**. Earlier we said suppose $1/N$ of the total body weight is supported by each of N wheels rolling. But remember that the distribution of a fixed amount of weight amongst the supporting axles/wheels really does not affect the overall frictional drag. This was shown in Lecture 11, where a car with from 6 to 3 wheels rolling still had the same frictional drag. What happens here is that when the *same* net weight must be carried by 3 instead of 4 wheels, the pressure of the axles on the bores of the 3 wheels increases their frictional drag just enough to keep the total drag constant. But when a 4th formerly self-supporting wheel is raised it adds to the net weight that must be supported by the other 3 and thus increases net frictional drag (One website claims a raised wheel causes less overall friction).

Also, note that the denominator in Eq (13) is larger than the denominator in Eq (18). This denominator represents the inertial effects of mass that tend to overcome drag deceleration. As an example, a gallon bottle 50% full of water will fall slower in air than the same bottle full of water. As shown in Lecture 1b, the less mass means more air deceleration. Because the wheel rotation inertia also counts as mass inertia, its addition also increases deceleration, here by 1.8%.

Fig 10 shows deceleration for a 4-wheel and a 3-wheel car in 3 different cases. The bottom comparison shows the net initial deceleration at the start of coast as just discussed in **Table 1**. Also shown is the case where the air drag coefficients CW, CB are set to zero but the friction coefficient is left at 0.1, and the case where CW, CB are left at 1.0 and the friction coefficient MU is set to zero. The 1.0 air drag is about twice as much as on an ordinary PWD car, and represents just an unstreamlined square body block. The curves were generated in an Excel spreadsheet by using the new [Display Results] feature in VR-II that contains both tables and graphs of any single car virtual race. The acceleration during the 16 feet or so of ramp travel starts out pretty high at about +450 cm/s^2. Both cars have the same potential energy at the race start, but the 4-wheeled car must store more of this energy as rotational compared to the 3-wheeled car. So the larger translational energy of the 3-wheeled car will put it ahead at the end-of-ramp (EoR). But as soon as the coast starts, the stored rotational energy begins to be converted to translational. The 4-wheeled car has more stored rotational energy and the coast advantage of less deceleration and can thus eventually overtake the 3-wheeled car.

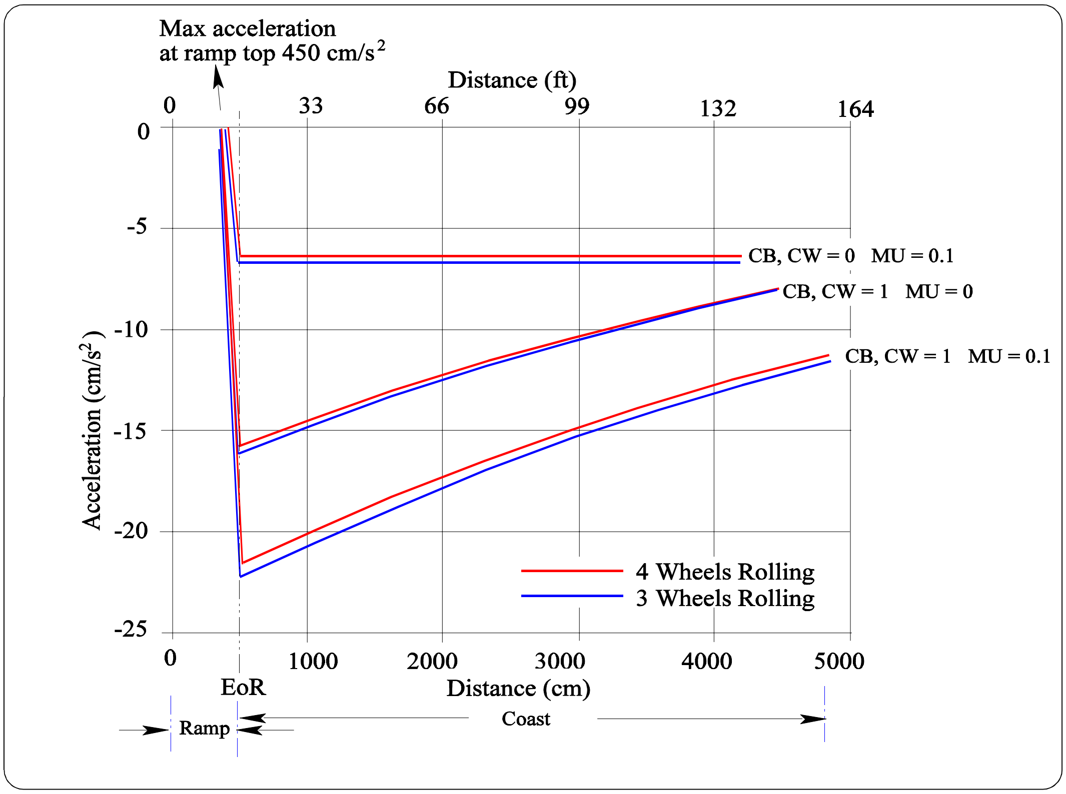


Fig 10 - *Using the [Display Results] feature of VRII and Excel to examine coast accelerations.*

In practice, it is unlikely that any 4-wheel car can have its weight continuously supported by 4 wheels. The coasting track surface is not flat to within a few thousandths of an inch, and at any given instant the car will be supported momentarily by only 3 wheels according to the plane they determine. Because of usual rear wheel weighting, the two rear wheels will always be in firm rolling contact. The two front wheels will thus alternate between which one carries the front load depending on the flatness of the track encountered. But, for all practical purposes, the momentary contact of one or the other front wheels will keep the rotation of both about the same as if they continually touched.

The 4-wheel car overtaking the 3-wheel car is based on the same physics as a heavy-wheeled 3-wheel car overtaking a light-wheeled 3-wheel car. So the lighter wheels are in great demand. But they don't always provide an advantage. The Crossroads of America Council of the BSA during a Spring 2009 event advertised "The Indiana State Museum's Fantastic Pinewood Derby Track Will be Ready and Rigged for Action. It is 120 feet long!" There was some confusion there when the light-wheeled cars suffered defeat when racing against ordinary heavy-wheeled cars.

In this lecture, some applied physics and calculation math detail was included so the reader could appreciate that the little Pinewood Derby car physics can be fairly complicated, yet the Virtual Race program does literally hundreds of calculations more complicated than this every few microseconds when it is running.