Cub Lecture 2 - Understanding Sliding Friction

Sliding friction is the resistance an object has to being moved when you try to slide it across the surface of another object. In this lecture/quiz we will look at experiments demonstrating friction by sometimes repeating parts of the experiment and gradually increasing the required level of understanding. So if we go too slow for you, bear with us, others may not be so sharp.

Distance and areas

Have a look at **Figure 1**. At the bottom left, inset **a,** we see a ruler marked off in fractions of an inch, a part of the old English units system. The smallest length we see is 1/16 in ("in" is short for inch, with the period after the abbreviation removed so as not to be confused with a decimal point). See the question mark **?** between the 1/4 in and the ½ in mark ? Circle the correct answer letter on this **question 1**:

a 3/16
b 3/8
c 5/16
d 9/8
e 7/16

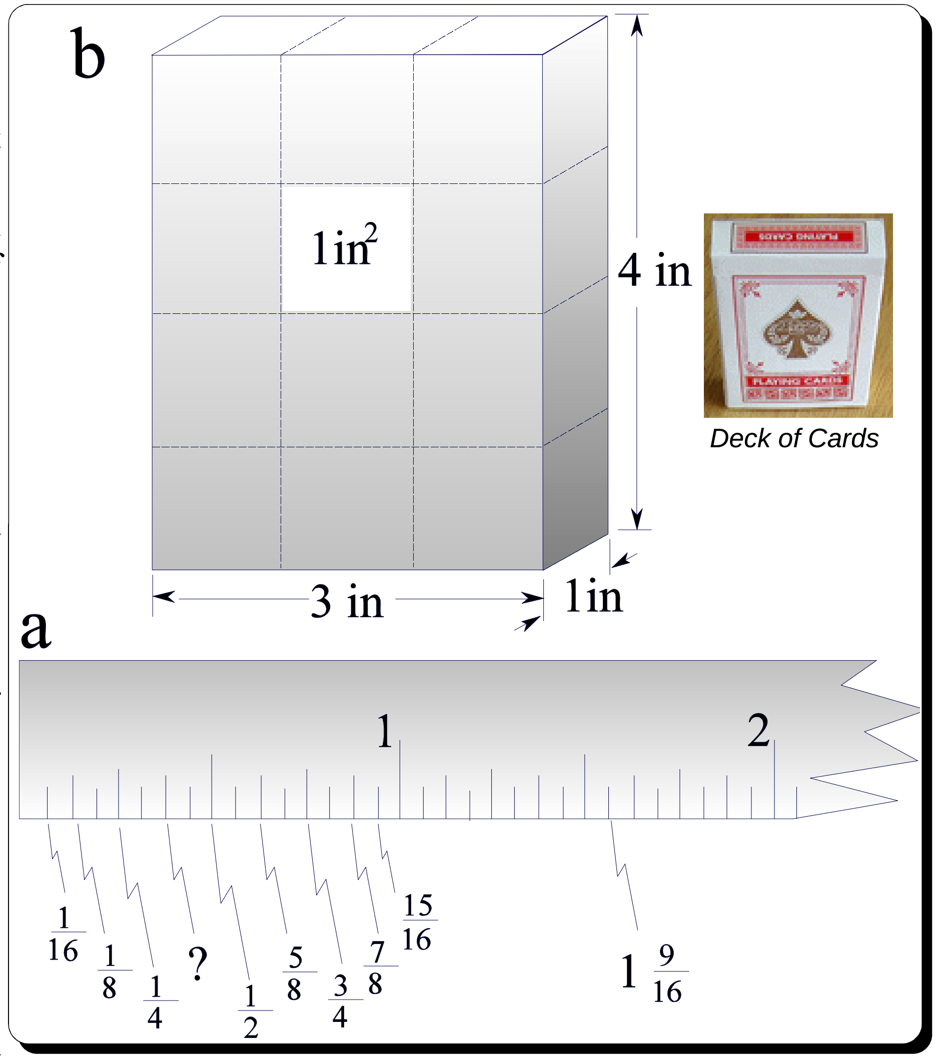


Figure 1 *- Using a ruler to measure lengths and areas.*

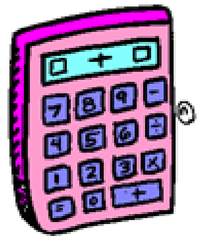
At the top left, inset **b**, is a rectangular block about the size of a deck of cards. Its sides are surfaces called areas, and each area has a length and a width. To get areas you multiply length times width. A piece of the largest block side, shown in white, with length 1 in and width 1 in would have an area of 1 in x 1 in = (1 x 1) x (in x in) = 1 in^2. So when you multiply distances to get areas, you need to first multiply the numbers, then the units, getting one square inch where the square may be written as a 2 in the superscript (smaller, higher, and to the right) position.

In **Fig 1b**, circle the correct answer letter for the area of the block's entire largest side as:

question 2:
a 10 in^2
b 12 in
c 12 in^2
d 5/16 in
e 14 in^2

Oh yes, you may print this lecture/quiz on paper, but <u>for your use only</u> because it's copyrighted. Print out the answer sheet Word document to keep track of your answers.

The English units can give us calculation problems because they are fractions, which means something (numerator) divided by something (denominator). And if you're like me, you hate to multiply and divide or even add and subtract fractions. Lets go ahead and do the division once and for all so we can just use the quotients, called decimal equivalents. **Figure 2** shows these decimal equivalents in the center column. Grab your calculator and check a few of these. For 5/16, enter a 5, hit the divide key and then enter 16 and hit the = key. You will get 0.3125. Now just .3125 is also correct, but the decimal point (a period) is pretty small, so if you are reading fast or in a hurry, you may miss seeing the decimal. Thus, you may think 3125, which is a big number. For decimal numbers less than 1, engineers and scientists have gotten into the habit of putting a "0" to the left of the decimal. It doesn't change the value of the small decimal number, and it's a lot bigger than just a ".", and thus the leading "0" means that, "Hey!", there's a "." following me, so don't miss it (or you could be three thousand times too big). Sometimes it doesn't matter, but say a big mouse jumped on your head that weighed about a third (.3333) of a pound, so this would be a lot easier on you than a 3333 pound mouse landing on your head. So let's make that mouse weigh in at precisely 0.3333 lbs.



English	Decimal	Rounded to 2 dec. places
$\frac{1}{16}$	0.0625	0.06
$\frac{1}{8}$	0.125	0.13
$\frac{3}{16}$	0.1875	0.19
$\frac{1}{4}$	0.25	0.25
$\frac{5}{16}$	0.3125	0.31
$\frac{3}{8}$	0.375	0.38
$\frac{7}{16}$	0.4375	0.44
$\frac{1}{2}$	0.5	0.50
$\frac{9}{16}$	0.5625	0.56
$\frac{5}{8}$	0.625	0.63
$\frac{11}{16}$	0.6875	0.69
$\frac{3}{4}$	0.75	0.75
$\frac{13}{16}$	0.8125	0.81
$\frac{7}{8}$	0.875	0.88
$\frac{15}{16}$	0.9375	0.94
1	1	1.00

Figure 2 *- Decimal equivalents to fractions and rounded values.*

When we start measuring something here in a moment with the ruler, remember our eyes are just so good. Look at the edge of one of the playing cards. The thickness of this card is about 0.010 in. This is read as one ten thousandths of an inch or one hundredth of an inch. The whole stack of 52 cards is just over ½ in = 0.5 in, and 52 x 0.010 = 0.52 in. Thus, about one hundredth of an inch is about as small as we can readily see. Our eyes would need help, with a magnifying glass or a micrometer caliper, to see only one or 2 thousandths of an inch like the thickness of a sheet of ordinary paper. So lets just get rid of all the decimal places that are less than the hundredths place. Notice in **Figure 2** the rightmost column has only two decimal places. Also notice that we didn't just throw away the 3rd and 4th decimal place numbers of the center column. If the 3rd place number was smaller than 5, we threw it and any 4th number following away. But if it was 5 or larger, we made the 2nd place number go up one. This is called rounding to the hundredth place. Now the numbers like 0.0625 aren't wrong, but they are misleading if we report them! Because writing down 0.0625 inch implies that we can actually see that well, but with our unaided eyes the 3rd and 4th places are just not *significant*. So the rightmost column figures are all now significant to the 2nd place.

A number like 0.5 is not significant to the 2nd decimal place because it has no second decimal place. If we can see something to within a hundredth of an inch, then we should report it, so we should write it as 0.50 inch unless we see the measurement might be somewhat smaller than ½ inch as in 0.49 inch or somewhat larger as in 0.51 inch.

This brings us to **question 3**. How many of the 16 numbers in the center column of **Fig 2** are significant to the 2nd decimal place?

question 3:
a none
b one
c two
d three
e four

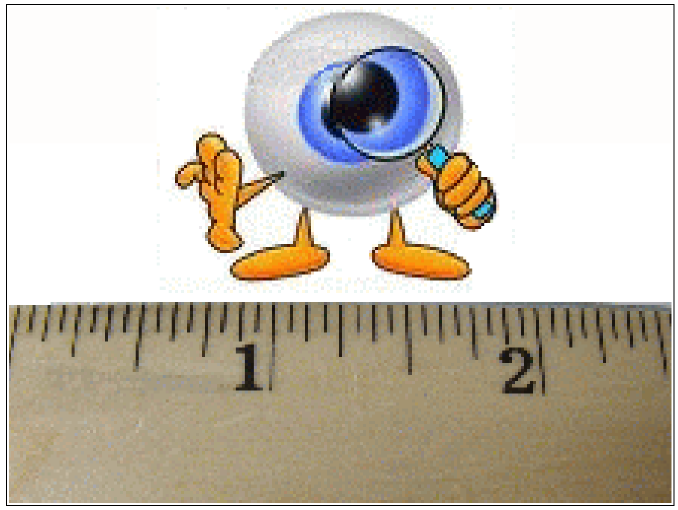
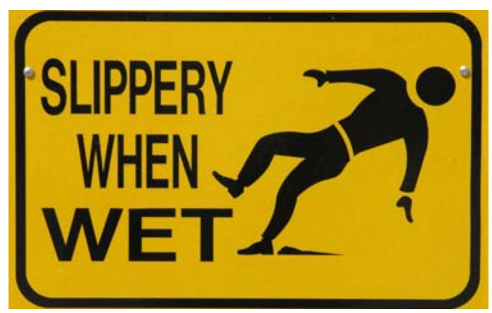


Figure 3 *- Remember, numerical significant figures are what we can see are really there, no more, no less.*

The Scientific Method

We will used a procedure called the Scientific Method to determine how sliding friction affects things. A detailed version is in **Lecture 3**.

In the *Scientific Method* (hypothesis first):



First, we guess how something is supposed to act based on just thinking about it, or perhaps on our experience sometimes called "common sense". This guess is then stated as a "hypothesis" which means something we think will happen but will nevertheless need to be proved or disproved through observation and experiment.

Second, the scientific part means the "method" must be based on gathering real evidence that can be subjected to specific principles of reasoning, usually through the logic of mathematics. So our reasoning could use calculations, like algebra, or proofs, like in geometry.

So lets think about things sliding. Your shoe sole sliding across a tile floor might offer quite some resistance, but if you were walking on wet ice, the resistance could be much less. Offhand you might think "Well, if I drag something of a certain area across a flat surface, then its bottom area rubbing on top of the flat surface will cause some sort of resistance. Likely large if the surface is rubber and likely a smaller resistance if the surfaces are wet. And, most people would agree, if there is more surface involved in the sliding, then the resistance to the sliding must be more.

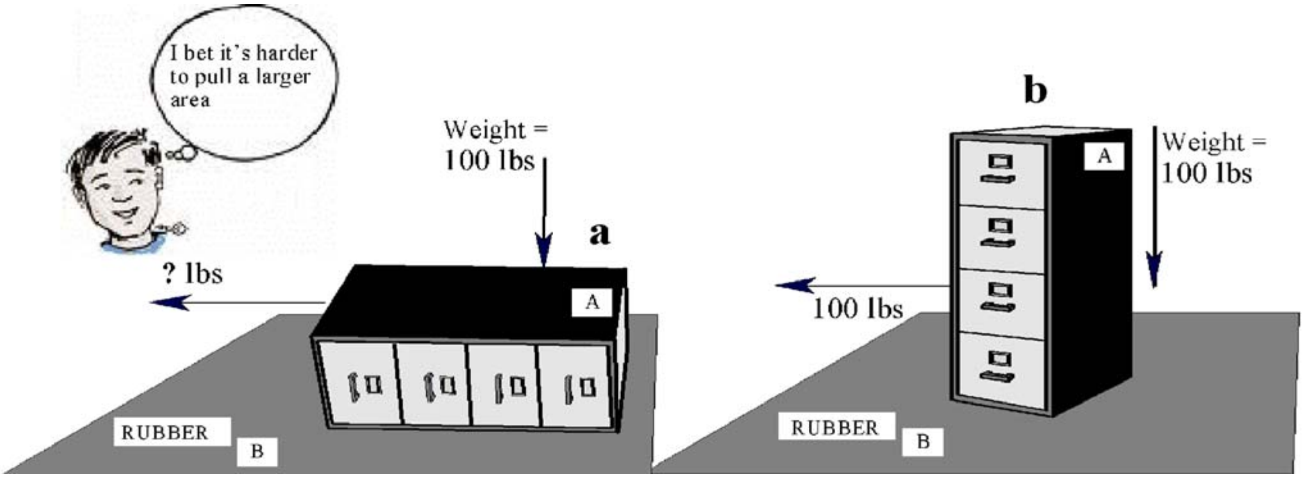


***Figure 4** - Pulling on a cabinet to make it slide across the floor, large area **a** small area **b**.*

But look at **Figure 4**. Suppose you were trying to pull a flat-bottomed filing cabinet A over a floor B covered by a smooth non-skid surface (like the rubberized paint for a pickup truck bed). If the filing cabinet weighed 100 lbs, you might have to pull with a horizontal force of 100 lbs as in **b** to get it to slide. If the cabinet were put on its side as in **a**, a much larger area would be in contact with the floor, and you might think it would take more pulling force. After all, the friction comes from the object's bottom surface sliding on the floor, so for more bottom surface, the harder it should be to slide. Thus, the frictional drag would be more. For a hypothesis we might have (as a lot of people believe):

Hypothesis 1:
The frictional drag force experienced when one object slides on another increases as the contact area increases, even if the weight of the sliding object stays the same.

Measuring Areas

Let's measure the areas of the sides of the playing card boxes. We have 3 sides, a large, medium, and small. Unlike the block example in **Figure 1b,** the card deck is not whole inches. I'll measure the large area of a card deck as an example, using a space to separate the whole number from the fractions. A(large) will represent the Area of the largest side.

A(large) = 2 1/2 in x 3 7/16 in = 2.50 in x 3.44 in = 8.60 in^2.

Now you measure the areas of the other smaller sides, called the medium and small areas.

A(medium) =_______ in x _______ in = _____ _in x ______ in = ______ in^2.

A(small) =_______ in x _______ in = _____ _in x ______ in = ______ in^2.

In **Fig 1b**, the block would give A(medium) = 4.00 in^2 and A(small) = 3.00 in^2
(Notice everything is to 2 significant decimal places)

question 4: Your A(medium) is between which of the following two in^2 numbers, where < means the number to the left is less than the number to the right, e.g., 1.50 < 2.05 < 2.75 ?

a 0 < A(medium) < 1.19
b 1.20 < A(medium) < 2.19
c 2.20 < A(medium) < 3.19
d 3.20 < A(medium) < 4.19
e 4.20 < A(medium) < 5.19

question 5: Your measure of A(small) is between which of the following two in^2 numbers:

a 0 < A(small) < 1.19
b 1.20 < A(small) < 2.19
c 2.20 < A(small) < 3.19
d 3.20 < A(small) < 4.19
e 4.20 < A(small) < 5.19

Dragging the full card deck boxes (sometimes called just "deck")

Tape the corners of a plain white piece of ordinary computer printer paper onto a table or cabinet top. Then cut the loop of a No 19 rubber band. Tie a basic overhand knot and pull it tight, making two knots close to opposite ends of the rubber band. This makes it easier to handle. You may use a new unopened deck with a clear cellophane-like covering, or an opened deck will be OK if it's not too used. Now tape one end of the rubber band as shown at the end of the deck as in **Figure 5**. The rubber band will stretch as we apply a force. We will assume that if we double a force we will double the amount of stretching of the rubber band. This is pretty close, but a fine-wound tension spring would be used for really precise force measurements.

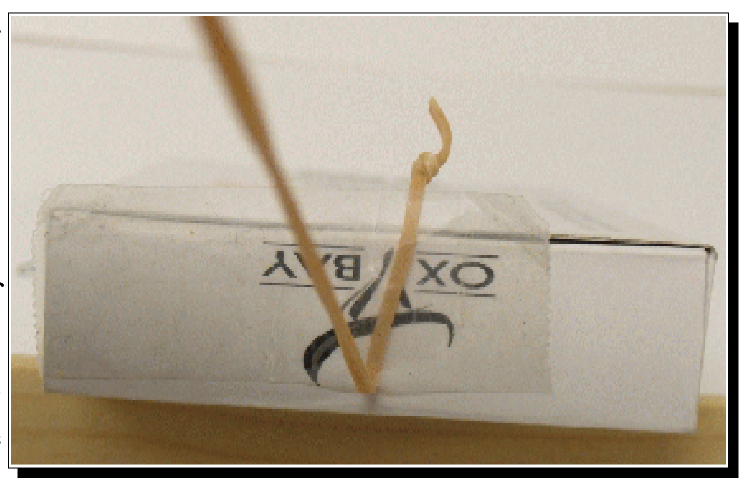


__Figure 5__ - Tape the rubber band to within 1/8 inch of the deck bottom, so it's not rubbing on the table when the deck is pulled horizontal as in __Fig 6__.

Now the next step is shown in **Figure 6**. We are going to slide the deck on it's A(large) surface. Pull the rubber band to the right until it just lifts from the sheet of paper you taped on the table top. **Figure 6** shows the rubber band being pinched at 4 in just as the rubber band has cleared the paper sheet. Then pull slowly to the right until the deck slides forward. At this exact time note where your thumbnail is next to the ruler. Do this 3 times, making a note of the distance each time, such as:

1. DistLargeArea = 0.94 in
2. DistLargeArea = 1.06 in
3. DistLargeArea = 0.81 in

AvgDistLargeArea = 0.94 in

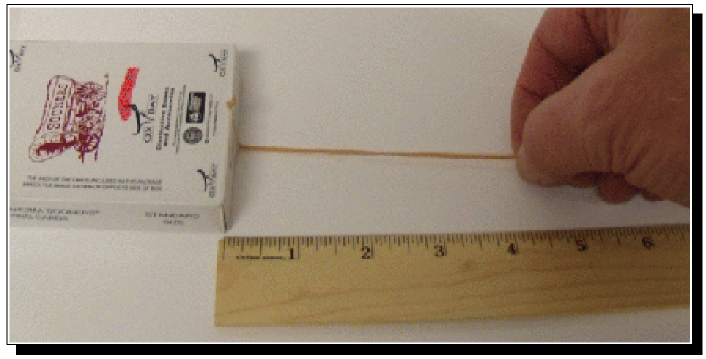


Figure 6 - Pulling the large area with a No 19 rubber band starting at 4 in. and measure distance over 4 in for a deck slide to start. Do it 3 times. Keep rubber band parallel to paper as you pull.

The distance to start a deck slide might be 4 15/16 in, so if you start at 4 in, the 1st stretch distance is 4 15/16 - 4.00 =15/16 in net stretch. **Figure 2** tells us this is 0.94 in. Suppose the second time your stretch the rubber band you get 1 1/16 in, and the 3rd is 13/16 in. Convert to decimals, add all 3 numbers, divide by 3, and you have the average stretch for DistLargeArea .

question 6: For an average DistLargeArea , what is your net stretch distance, in inches?

a 0.20 < AvgDistLargeArea < 0.59
b 0.60 < AvgDistLargeArea < 1.19
c 1.20 <AvgDistLargeArea < 1.59
d 1.60 <AvgDistLargeArea <2.19
e 2.20 <AvgDistLargeArea < 2.59

Figure 7 - Pulling the medium area and getting the average of 3 "stretch" measurements. Pinch thumbnail at 4 in just as rubber band lifts clear of paper,

question 7: For an average Distance Medium Area, what is your net stretch distance, in inches?

a 0.20 < AvgDistMediumArea < 0.59
b 0.60 < AvgDistMediumArea < 1.19
c 1.20 <AvgDistMediumArea < 1.59
d 1.60 <AvgDistMediumArea <2.19
e 2.20 <AvgDistMediumArea < 2.59

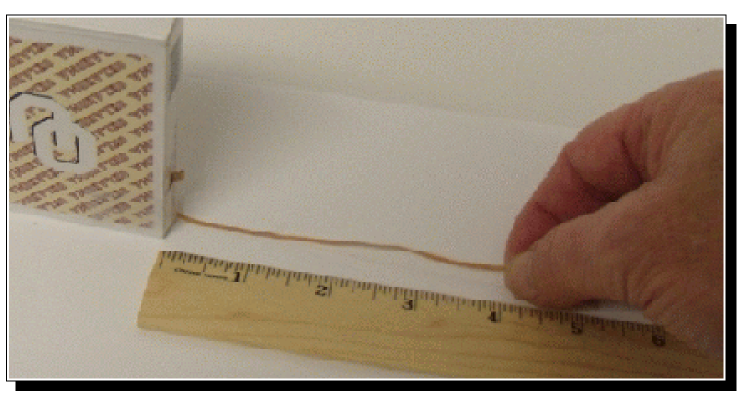


Figure 8 - Pulling the small area and getting the average of 3 measurements. Rubber band here may be just a hair too loose.

question 8: What is your Average Distance Small Area stretch, in inches?

a 0.20 < AvgDistSmallArea < 0.59
b 0.60 < AvgDistSmallArea < 1.19
c 1.20 <AvgDistLargeArea < 1.59
d 1.60 <AvgDistLargeArea <2.19
e 2.20 <AvgDistLargeArea < 2.59

Now comes the big **question 9**. The areas which you were sliding on the sheet of paper changed a lot if you were careful with your edge measurements.

question 9: So, did your force as measured by the rubber band stretch change a lot as well?
a The force to cause sliding, the stretch distance, was about the same for all 3 areas
b The forces to cause sliding were about 4 times greater for the large area than the other areas
c The forces to cause sliding were about 2 times greater for the large area than the other areas
d The sliding forces were about 2 times smaller for the large area than the other areas
e The sliding forces were about 2 times greater for the medium area than the other areas

question 10: Based on your answer to **question 9**, is the **hypothesis 1** on p 4 correct?
a Yes, it agrees with experiment, and perhaps should be part of a physics law
b No, it does not agree with experiment, and its opposite should be part of a physics law
c Sometimes
d Can't tell, not enough experiments were done
e I'm not sure, but seems like a yes, or perhaps it could be a no

Lets look at weight

We need to measure the force on the solid mass that gravity causes. This force is its weight. As in **Fig 9**, tape the small area so the rubber band will pull from the center, not 1/8 in from an edge. Lay the deck on the ruler with its top at the 5-in mark. Pinch with your thumb at the 1-in mark so the rubber band part under the tape can't stretch. Then lift the ruler as in **Fig 10** until the deck clears the bench top supported only by the rubber band. Make sure the band doesn't stretch under the tape.

question 11: The rubber band stretched how much to support the weight of the deck? If its top was at 10 in, in **Fig 10**, then 10 - 5 = 5 in stretch on the rubber band. Circle what you got closest to:

a 4.00 in
b 4.50 in
c 5.00 in
d 5.50 in
e 6.00 in

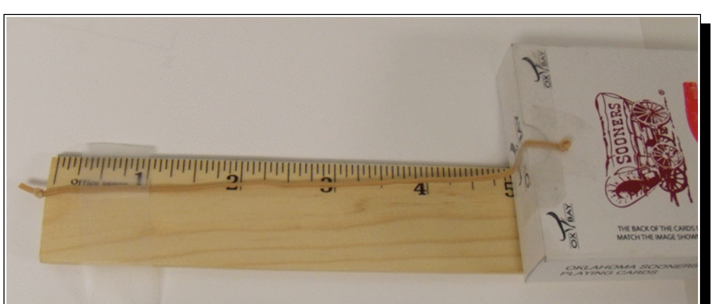


Figure 9 *- Prepare to weigh by taping to the deck small area center, set the deck top on 5 in mark, and tape the rubber band top at 1 in.*

Figure 10 *- Pinch the rubber band on the 1 in mark and raise the ruler until the deck starts to raise. Here the deck bottom has just cleared the bench top. Note the stretch distance on the ruler. Make sure the rubber band is a # 19.*

The coefficient of friction

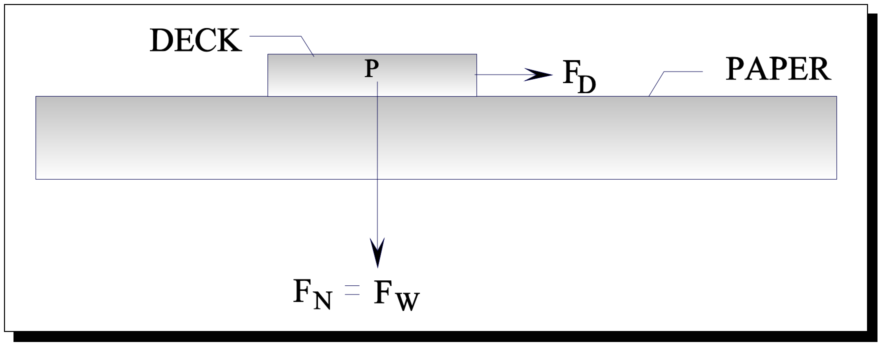


Figure 11 *- The forces on the deck sliding horizontally*

The coefficient of friction is the force required to slide a smooth object over a smooth surface divided by the perpendicular force on the object. See **Figure 11**. Here we let the symbol F_N stand for Normal Force, meaning the force on the deck perpendicular to the paper surface. With a horizontal paper surface, this normal force is also the Weight Force F_W. F_D is the Drag Force required to slide the deck parallel to the surface. The coefficient of friction is represented in physics by the Greek letter μ, pronounced "mew". So μ is defined as:

$$\mu = \frac{F_D}{F_W} \qquad \text{(Eq.1)}$$

Remember when I got 0.94 in for the drag force sliding the large area? And I got 5.00 in for the deck weight force also as measured in "inch stretch units"? So I would get for μ

$$\mu = \frac{0.94 \text{ in}}{5.00 \text{ in}} = \frac{0.94}{5.00} = 0.19$$

Notice that the "in" cancels out since both the numerator and denominator have it. So it doesn't matter what force units you use because as long as you keep them the same they will cancel. So μ has no units. Your calculator will give 0.188 but because we're putting into the calculator numbers that are only accurate (significant) to 2 decimal places the answer is only good to 2 decimal places as well. Remember with computers (and calculators) the old saying about "garbage in, then garbage out". So we should round off 0.188 to 0.19.

Put your answer from **question 6** into the calculator and divide by the number you got to answer **question 11.** Then answer **question 12** below: Note you may get an answer somewhat different from μ = 0.19. That's OK. Your paper could be a little different, your card deck could have a different finish, you may have higher or lower humidity, etc.

question 12: Your μ is closest to: **a** 0.10 **b** 0.15 **c** 0.20 **d** 0.25 **e** 0.30

More sliding friction

You don't have to do this experiment as a yard-stick wasn't on the equipment list, so there will be no questions. But please slide your deck on paper or on anything at an angle just for fun. Tape a sheet of paper to a yardstick as shown in **Fig 12**. Place a deck of cards on the paper.

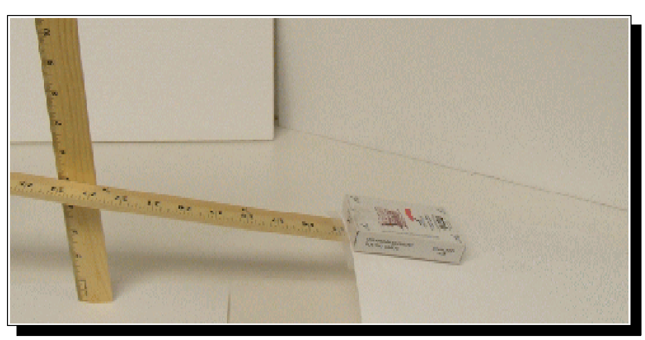


Figure 12 *- Deck on an an incline.*

Raise the left end of the yardstick while holding the 12 in ruler perpendicular to the table at the 24 in mark on the yardstick. Raise the left end of the yardstick until a deck of cards will just begin to slide down the paper. **Video 1** shows the deck sliding at a certain angle. **Figure 13** shows what's going on. Instead of the full weight force, F_W, being normal to the surface, this time we have F_W split into two forces, F_N normal and F_D trying to slide the deck. Look at insert **b**. The drag force F_D might as well be pulling along the surface at the center point P of the deck since the deck is a rigid body. This gives the shaded force triangle **b**. Now flip **b** over and rotate it and put it at position **c**. These force arrows are called vectors and have both a length and a direction. You can break up F_W into F_D and F_N using that old Greek fellow, Pythagorus', theorem. Because the **c** triangle is a right triangle with angles like in **a**, triangle **c** forces are proportional to triangle **a** sides. We can use Eq. 1 to get the coefficient of friction which turns out to be RISE/RUN.

Video 1. *Click left to see deck slide*

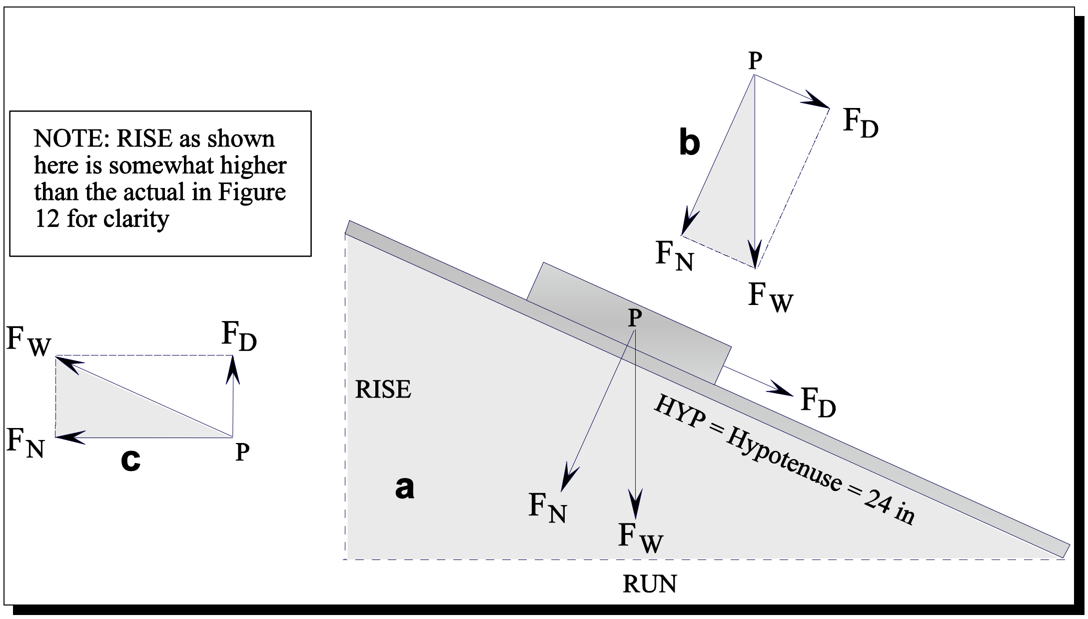


Figure 13 *- The forces on the deck when sliding down an inclined plane.*

Figure 14 shows that RISE = 4 9/16 = 4.56 in. Pythagorus says $RUN^2 = HYP^2 - RISE^2$, so RUN is square root of $(24.00^2 - 4.56^2) = 23.56$. Thus,

$$\mu = \frac{F_D}{F_N} = \frac{\text{RISE}}{\text{RUN}} = \frac{4.56}{23.56} = 0.19$$

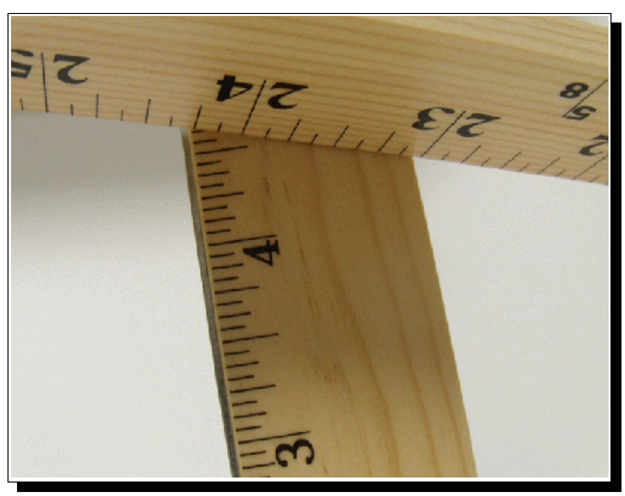


Figure 14 *- Measure RISE at HYP = 24 in*

It's pretty cool when another method gives the same value for the coefficient of friction.

Increase and decrease deck weights

My experience leads me to suppose that if the vertical force of gravity is doubled, then the force required to slide it across a surface will go up. How much? Lets suppose further that the frictional drag force itself is also doubled, so that:

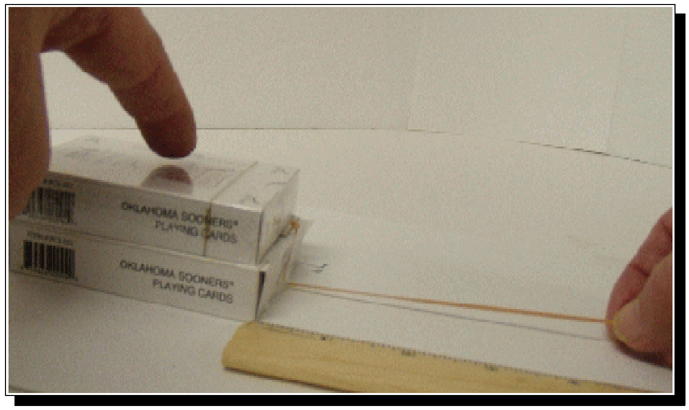


Figure 15 *- Slide two decks.*

Hypothesis 2:

The frictional drag force experienced when one object slides on another increases proportionally as the perpendicular force exerted by the sliding object increases.

So, do as you did in **Figure 6** and question 6, except here add the second deck on top of the first deck and again stretch the 4 in length of No 19 rubber band until the doubled deck slides. See in **Figure 15** that the rubber band is attached to the bottom deck just as in **Figure 6**. Notice my left finger is poised over the decks. You should tap gently on the top deck as you stretch the rubber band. This is because as weight force goes up, there is an adhesive force that builds up between surfaces when objects are stationary. It's sort of like a little bit of sticky material is present between the bottom surface and the sheet of paper. Now once sliding starts, this adhesive force goes away. As a result, the force it takes to start sliding may be a little more than the force required to sustain sliding.

A couple of definitions are helpful here:

Kinetic - Based on a Greek word that means having motion.

Static - Based on a Greek word that means not having motion.

Thus, the friction to start sliding is measured by a static coefficient of friction, and the force to sustain sliding is measured by the kinetic coefficient of friction. When you have just the weight of one deck, the static and kinetic friction coefficients are pretty close. But as the weight increases the normal force, a little tap is needed to jiggle those molecules on the bottom deck that are touching the molecules on the surface of the paper. This keeps the molecules from getting entangled and causing the extra static adhesive force.

question 13: The amount of rubber band stretching required to slide (with tapping) 2 decks compared to your amount in question 6 was

a about the same
b about half as much
c about twice as much
d about 3 times as much
e the rubber band broke

This next experiment may seem kind of silly. But, sometimes being silly is fun, and we are all for having fun! Try to drag only *one* card with the rubber band, as in **Figure 16**, and you see the card slide before you can even straighten the rubber band. The card weight is tiny, less than 2 hundredths the weight of the whole deck and its box (The amazing thing is, if you do like **Video 1**, one card slides at the same angle as the whole deck slid, meaning μ hasn't changed although weight went up x 52).

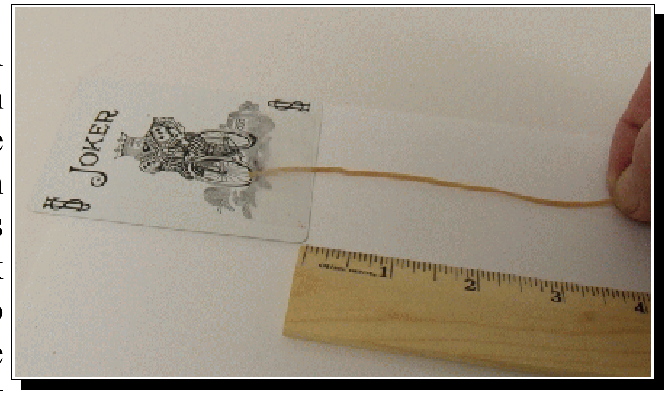


Figure 16 *- Slide one card*

Lets see how small a card weight is significant. I have in my lab an electronic digital scale that is accurate to 0.001 grams. The deck plus box weighs 87.498 grams and 1 card weighs 1.549 grams, only 0.018 times as much (the calculator gives 0.017703261 but remember the digital scale accuracy is only accurate to the 3rd decimal place so round off to 0.018). Thus, our F_W went down from 1.00 deck to 0.018 deck. This is pretty close to the 0.01 we had been using as the least significant figure length measurements. Also, the drag needed to pull the card would be even smaller than 0.018 of a deck weight. So within our measurement accuracy we can reliably say both F_N ($= F_W$) and F_D are close to zero for just 1 card sliding.

Let's make a physics law

Look at **Figure 17.** In the graph the normal force F_N is the deck weight, and F_D is how much force, or decimal fraction of the deck weight, is needed to slide it across paper. We just discussed the near zero point, and earlier we got $F_D = 0.19$ for $F_N = 1.00$ deck force, and I got about double this for $F_N = 2.00$ deck forces. The slope of the line, which is the "rise" over the "run", is the unchanging coefficient of friction μ. It looks like hypothesis 2 is correct, and we have a

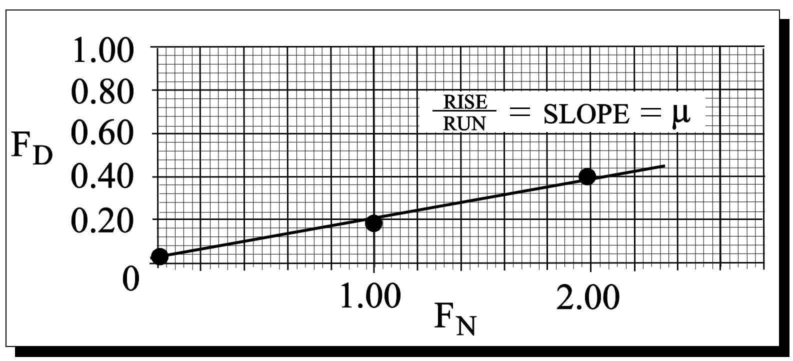


Figure 17 *- The drag force to slide a smooth object over another. F_N and F_D Units are Deck Weight.*

Law of Sliding Friction - *The drag force to slide a smooth object over another is proportional to the normal force that the sliding object exerts on the surface. The constant of proportionality is called the coefficient of friction.*

Moreover, the opposite of hypothesis 1 could be added to this law based on your experiments.

Law of Sliding Friction Corollary - *The coefficient of friction, and frictional drag, is independent of contact area in smooth sliding.*

question 14: The filing cabinet in **Figure 4b** would have what for a coefficient of friction?

a 0.19 **b** 0.40 **c** 0.60 **d** 0.80 **e** 1.00

Friction in wheels

We all love to roll toy cars, from a tiny tot age to older folks that race in the WIRL. Many wheels we use, like on automobiles, have ball bearings (or roller bearings) to really reduce energy lost through friction because such bearings have no sliding surfaces. The simplest bearing, though, is called a journal bearing. It's just a cylindrical axle, the journal, inserted into a cylindrical bore hole in the center of the wheel. Some journal axles, like on railroad cars, are cast as part of the wheel pair, and the cylindrical opening in which the axle rests is made inside a "box" casting. In the pinewood derby journal bearing, the old wagon-type simple fixed axle slides-inside-rotating-wheel bore is used.

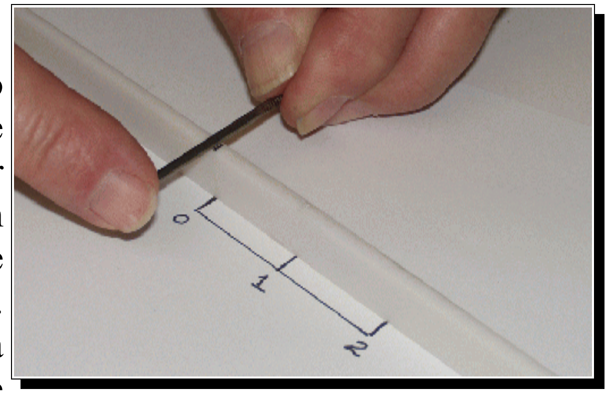


Figure 18 *- Sliding the 2-inch nail on a plastic strip 1/8 in wide and ½ in tall.*

In **Figure 18,** I hold the 2 in nail on top of a strip of plastic and push down as I rub the nail forward for 1 in on top of the plastic (You don't have to do this, but if you do, you can slide the nail down the length of a plastic ball point pen instead of a flat piece of plastic). I made the inch marks on the paper sheet with the ruler then removed the ruler so my fingers wouldn't bump it. Suppose I push down with a 1.00 ounce force (oz_F), which is about 1/4 the weight force of a pinewood derby car body. And, lets suppose the coefficient of friction for steel (the nail) sliding on plastic is 0.30 (it is). So the drag force as I slide the axle 1.00 inch is 0.30 oz_F. Now get this very important definition:

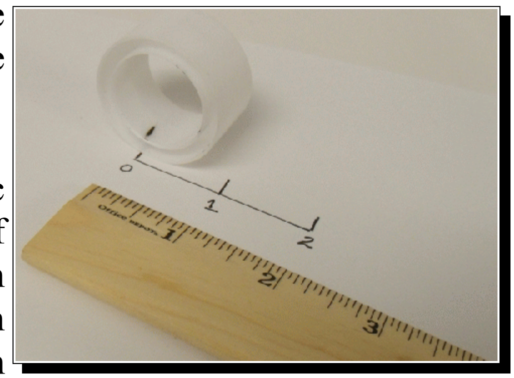


Figure 19 *- Marking the roller.*

The kinetic energy (KE) involved in motion is the force causing the motion times the distance through which such force acts. Thus, the kinetic energy lost to friction is:

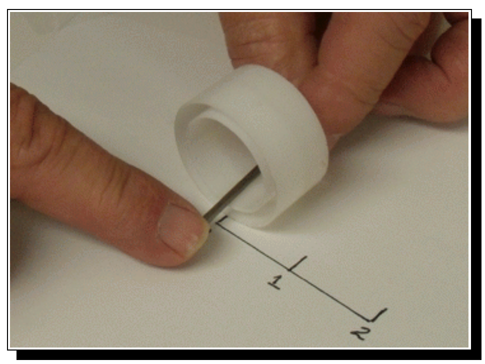


Figure 20 *- Start a 1 inch slide.*

$$KE = (1.00 \text{ in}) \times (0.30 \text{ oz}_F) = 0.30 \text{ in oz}_F \qquad \text{(Eq. 2)}$$

Now make a mark with the pen on the inside of the white plastic roller as in **Figure 19**. As in **Figure 20**, slide the axle forward as you push down with, play like, 1.00 oz_F of force. Slide the axle so it's right over the 1 in mark and make a 2nd mark inside the roller where the axle ended up as in **Figure 21**. Notice as the axle slides forward the roller bottom rolls along the surface, sort of a inner slide/outer roll combination.

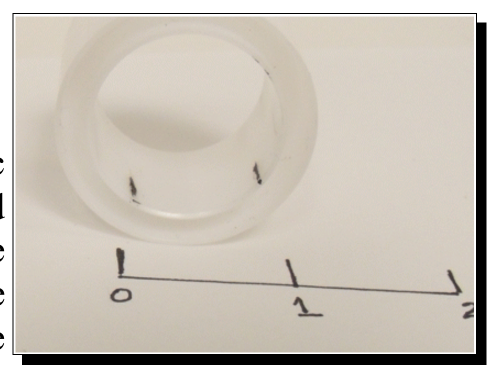


Figure 21 *- Mark the end of the 1 inch slide.*

The curved distance between the marks inside the roller is called an arc distance. It looks to me to be about 3/4 in straight distance. I'll give you a hand in measuring this arc distance by first measuring the roller inside diameter (ID) with precision digital calipers as shown in **Figure 22**. I get 1.0245 in which rounds off to 1.02 in. Now you can measure the outside diameter (OD) of the roller with the ruler and get the number:

Figure 22 *- Measuring roller ID as 1.02 in.*

OD = ___________ in Then consider these relations:

$$\frac{\text{INNER ARC DIST}}{\text{OUTER ARC DIST}} = \frac{\pi\ \text{ID}}{\pi\ \text{OD}} = \frac{1.02}{\text{OD}}$$

$$\text{INNER ARC DIST} = \text{OUTER ARC DIST} \times \frac{1.02}{\text{OD}}$$

$$\text{INNER ARC DIST} = 1.00\ \text{in} \times \frac{1.02}{\text{OD}}$$ (Plug in your OD distance in inches here)

In the calculations above, recall the curved distance is called arc distance, and the inner arc distance always has the same ratio to its outer arc distance as the total inner circumference has to the total outer circumference. Remember outer circumference = π OD in inches and the π's and inches in the numerator and denominator cancel. Also, the OUTER ARC DIST = how far you rolled (without slipping or sliding) which was 1.00 inch (think about why this is).

question 15: Your number for the INNER ARC DIST, in inches, for the plastic roller is closest to:
a 0.22 **b** 0.40 **c** 0.50 **d** 0.75 **e** 1.20

In **Figure 23**, roll a complete new tape roll of Scotch™ magic tape as you just did using its inside plastic roller. Measure its outer diameter, OD, and use

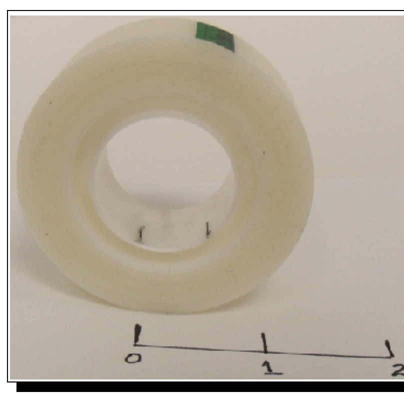


Figure 23 *- Marking the roller inside for rolling an entire roll of tape 1 in.*

$$\text{INNER ARC DIST} = \text{DIST ROLLED} \times \frac{1.02}{\text{OD}} \quad \text{(Eq. 3)}$$

to get the distance the nail axle rubbed in the inside hole, namely, its INNER ARC DIST.

question 16: Your number for the INNER ARC DIST , in inches, for the distance the nail rubbed the inside hole surface while rolling the entire tape roll, is closest to:
a 0.22 **b** 0.40 **c** 0.50 **d** 0.75 **e** 1.20

Remember, the kinetic energy lost to friction when the axle nail rubbed on straight plastic for one inch nail travel, carrying a 1 oz_F load was Eq. 2, which we repeat below:

$$KE = (1.00 \text{ in}) \times (0.30 \text{ oz}) = 0.30 \text{ in } oz_F$$

Say we used a plastic roller that had an ID over OD ratio of 0.75? Then when we pushed the nail 1.00 inch, the distance the nail slid on plastic while loaded with 1.00 oz_F weight would be reduced, right? Moreover, as in Eq. 3, we just showed that the inner circle arc distance axle slide compared to the straight axle travel distance while rolling goes as the ratio:

$$\left(\frac{ID}{OD}\right)$$

So the KE lost to friction when the axle goes forward 1 inch while rolling for 1 inch is:

$$KE = (1.00 \text{ in}) \times \left(\frac{ID}{OD}\right) \times (0.30 \, oz_F) \qquad \text{(Eq. 4)}$$

question 17: Use Eq. 4 to first calculate the KE lost to friction for the plastic roller rolled 1 inch, followed by the KE lost for the whole roll of tape being rolled 1 inch. The units are understood to be in oz_F. Use the numbers you got for OD in questions 15 and 16, and use my ID = 1.02 in. Pick the numbers below closest to your numbers_________in oz_F, ________in oz_F:

a 0.15, 0.23
b 0.55, 0.50
c 0.23, 0.15
d 0.75, 1.40
e 012, 0.12

In what follows, watch photos of me playing with a pinewood derby car body. Look at **Figure 24.** Assume the weight pressing down on each of the four 2 in nail axles is 1.00 oz_F.

question 18: How much KE is lost if all 4 nail axles are sliding 1 inch on plastic instead of just 1 axle? Units are in oz_F. (Hint, remember Eq. 2)

a 0.15
b 0.55
c 0.23
d 0.75
e 1.20

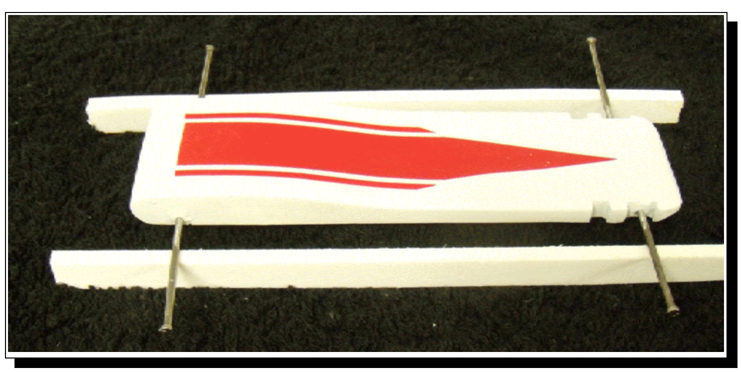


Figure 24 *- Pinewood Derby car, 4 axles sliding on plastic strips 1/8 in wide.*

Look at **Figure 25**: How much KE is lost if all 4 wheel axles are sliding in the inner (ID) hole to roll each roller forward 1.00 inch? Units are in oz_F. Your calculation gives _________ in oz_F.
(Hint, remember Eq. 4)
question 19: The number closest to yours is:

a 0.55
b 0.35
c 0.50
d 0.90
e 1.40

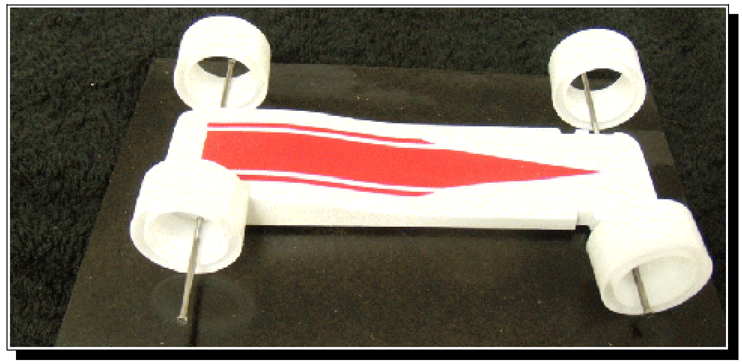


Figure 25 *- Pinewood Derby car, 4 axles sliding on the inside of plastic rollers of Figure 21.*

Look at **Figure 26**: How much KE is lost if all 4 wheel axles are sliding in the inner (ID) hole to roll each tape roll forward 1.00 inch? Units are in oz_F. Your calculation gives _________ in oz_F.
(Hint, remember Eq. 4)
question 20: The number nearest yours is:

a 0.55
b 0.35
c 0.50
d 0.15
e 1.25

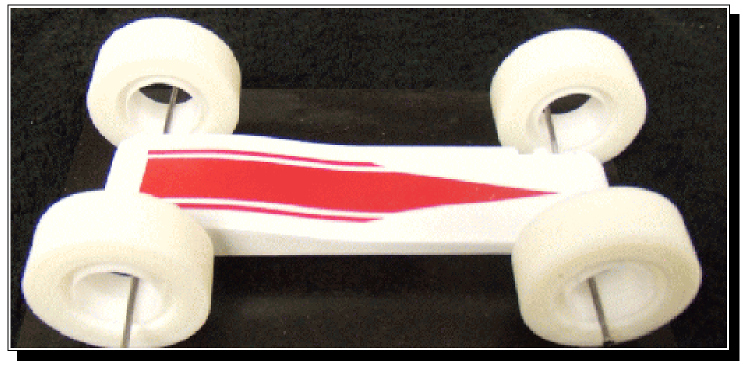


Figure 26 *- Here are 4 axles sliding on the inside of the same rollers as* ***Fig*** *25 but with a larger OD.*

Look at **Figure 27**: The wheels are painted white to show up well. We have switched the 2 in long nails to regular PWD axles of the same diameter.
How much KE is lost if all 4 wheel axles are sliding in the inner (ID) hole (called a bore hole) to roll the body forward 1.00 inch? We will use Eq. 4, and assume its numbers are good to 3 decimal places. The Wheel ID (bore diameter) = 0.096 in and the OD = 1.195 in. Units are in oz_F.

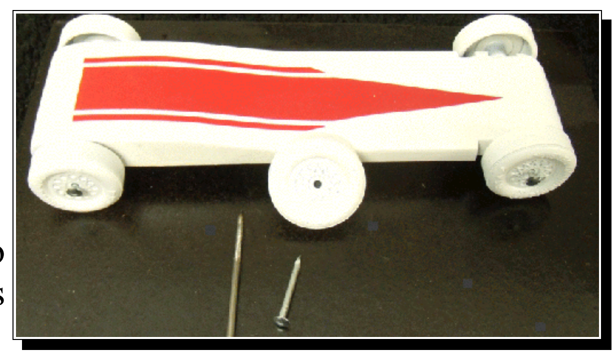


Figure 27 *- Pinewood Derby car, 4 axles sliding inside bores of regular 1999 wheels, ID = 0.096 in, OD = 1.195 in.*

$$KE = 4 \times (1.00\ \text{in}) \times \left(\frac{0.096\ \text{in}}{1.195\ \text{in}}\right) \times (0.30\ oz_F) = 0.096\ \text{in}\ oz_F$$

Before, (see **Fig 24**), with no wheels,
KE = 4 x (1.00 in) x (0.30 oz)
KE = 1.20 in oz_F.
So the wheels reduce sliding friction by 12.5 times, i.e., 1.20/0.096 = 12.5

Note: If the bore hole becomes very small as in **Figure 27**, and depending on how flat the supporting surface is, one front wheel (if car rear is weighted) may hang from the top of its axle. We are assuming this is not the case, the support is perfectly flat, and all 4 axles are coplanar.

The effect of friction on your race time

Now we have found how much KE is used in the sliding/rolling action of a PWD car as in **Figure 27**, lets see how much that can slow us down getting to the finish line. Remember Eq. 4 below:

$$KE = (1.00 \text{ in}) \times \left(\frac{ID}{OD}\right) \times (0.30 \text{ oz}_F) \qquad \text{(Eq. 4)}$$

And remember the 0.30 oz_F came from the perpendicular weight of 1 oz_F (on one wheel) times the coefficient of friction of plastic on steel, namely $\mu = 0.30$. Thus, the friction KE for going distance S in forward travel for all 4 wheels is:

$$KE = 4 \times S \times \left(\frac{ID}{OD}\right) \times \mu \times 1.00 \text{ oz}_F \qquad \text{(Eq. 5)}$$

So lets see how much KE is used when the car rolls a distance S = 16 ft. And let's say we use a good lubricant like oil (it doesn't hurt the wheels) or graphite. The lubricant could really reduce the coefficient of friction, so make μ six times smaller at $\mu = 0.05$. So Eq 5 becomes:

$$KE = 4 \times 16 \text{ ft} \times \left(\frac{0.096 \text{ in}}{1.195 \text{ in}}\right) \times 0.05 \times 1.00 \text{ oz}_F$$

$$KE = 4 \times 16 \text{ ft} \times 0.080 \times 0.05 \times 1.00 \text{ oz}_F$$

$$KE = 4 \times 16 \times 0.080 \times 0.05 \times 1.00 \text{ ft oz}_F = 0.26 \text{ ft oz}_F$$

This is the energy lost to friction, but you may wonder how much velocity is changed? And, you may wonder, how much will this friction slow you down at the finish line?

Well, you've done enough work measuring the friction that a pinewood derby car might have, so on the next page we'll let your senior partner, or perhaps your science teacher, help out. On this page, they perhaps can explain to you that your car, even with the very low $\mu = 0.05$, will still be about 1/3 of a car length behind a car with zero friction (on a 32 ft track).

Finally, after the next page, there comes the final exam for Pinewood Derby Car friction, Lecture 2. We will let you answer a few yes and no questions. These questions are ones that tend to confuse people when it comes to pinewood derby car friction. Remember your sliding friction law and corollary.

Happy Racing !

NOTE: The Cub or other youngster is not responsible for the contents of this box. But it would be nice if their senior partner could help in getting a few of the key concepts (**bold**) across to them.

First, lets **get rid of the x every time we multiply** things together. So if mass = M and velocity = V, then mass times velocity is MV. Another way is to write it is use parenthesis to enclose the things being multiplied together. So M x V = MV = (M)(V). Occasionally, but rarely, a center dot "·" indicates multiply as in M x V = M · V. When units are multiplied together, like inches times ounces force, one could use the following: in-oz_F ; in · oz_F ; in.oz_F ; in oz_F . We use the latter, with just a space separating the units symbols.

Second, we found the kinetic energy used in sliding friction was $KE_{FRICTION}$ = 0.26 ft oz_F. We want to compare this against the **kinetic energy of the whole PWD car**, its energy of free motion, which is:

$KE_{PWD} = \frac{1}{2} MV^2$ This free motion will have less velocity if friction converts some of the energy to heat.

This kinetic energy depends directly on the actual *mass* in ounces, not the force with which gravity pulls down on one ounce of mass, such force called oz_F . Suppose the whole car, body plus wheels, has a mass of 5 ounces, where we use the symbol oz_M to denote ounces meaning mass only.

When we push 5 oz_M horizontally, like rolling a small 5 oz_M bowling ball, it takes a certain horizontal force to get it up to some velocity, regardless of what force gravity is pulling down with (i.e., the 5 oz_F weight force).

Third, we use **Newton's law of gravity**, which relates just plain mass, 5 oz_M , to the weight of such mass, 5 oz_F. This law says that weight force = **mass** times the **gravitational acceleration g**. The gravitational acceleration, g , on the earth, is 32 feet per second per second, where per second per second is s^2 .

Thus $(0.26\ oz_F) = (0.26\ oz_M)g = (0.26\ oz_M)\left(\frac{32\,ft}{s^2}\right) = 8.32\ oz_M\ ft/s^2$

And, $KE_{FRICTION} = 0.26\ ft\ oz_F = 8.32\ oz_M\ ft^2/s^2$

Fourth, lets play like the PWD car rolls down a ramp for 16 ft, and there isn't any wheel friction on the ramp. If the car starts at 4 ft above the coasting run, its velocity V after it has dropped H = 4 ft is:

$$V = \sqrt{2gH} = \sqrt{2\left(\frac{32\,ft}{s^2}\right)4ft} = \sqrt{256\left(\frac{ft^2}{s^2}\right)} = 16\ ft/s \text{ (about 11 mph)}$$

$KE_{PWD} = \frac{1}{2} MV^2 = (\frac{1}{2})(5\ oz_M)\ 256\left(\frac{ft^2}{s^2}\right) = 640\ oz_M\ ft^2/s^2$ This is Kinetic Energy, no friction.

$KE_{NET} = KE_{PWD} - KE_{FRICTION} = (640 - 8)\ oz_M\ ft^2/s^2 = 632\ oz_M\ ft^2/s^2$ This is energy left after friction is subtracted, where we rounded off 8.32 to just 8. **Friction energy goes into heat**. The velocity after friction is:

$$V^2 = 2\left(\frac{KE_{NET}}{M}\right) = \left(\frac{1264\,oz_M\,ft^2}{5\,oz_M\,s^2}\right), \text{ thus } V = \sqrt{252.8\left(\frac{ft^2}{s^2}\right)} = 15.9\ ft/s$$

$$\text{Time difference} = \left(\frac{16\ ft}{15.9\,ft/s}\right) - \left(\frac{16\ ft}{16\,ft/s}\right) = 1.0063\ s - 1.000\ s = 0.0063\ s$$, (assume good timer),

or about 1/6 car length difference* at the ramp end. Over a 32 ft track, we would have about 1/3 car length difference at the finish line between a car with axle/wheel friction at $\mu = 0.05$ and a perfect car with $\mu = 0$.

*The time for a 7 inch car length to pass a line is $\left(\frac{7/12\ ft}{15.9\,ft/s}\right)$ 4 $0.037s$

Final exam - Car weight distribution and axle surface area

We will use your experiments to let you answer questions that confuse many PWD racers. One question is how does the frictional drag on a car change as the 5.00 ounces is redistributed among the wheels?

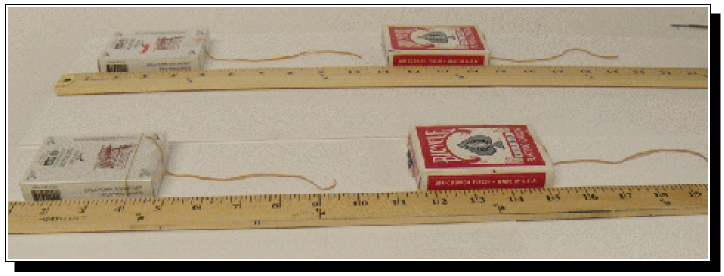


Figure 28 *- Sliding 4 decks of cards separately as done singly in Figure 6.*

Remember when you pulled a deck of cards with a rubber band to measure its sliding friction force? In **Figure 28** we show 4 decks of cards connected to rubber bands. So if you pull all 4 rubber bands at once, you would be applying 4 times the amount of force to cause sliding of one deck, right?

Figure 29 *- Moving the weight from the left rear to double the right rear deck weight.*

Now look at **Figure 29**. Remember **Figure 15** and **question 13**? By putting the weight of the deck of cards at the top left on top of the deck at the bottom left we would double the force needed on that rubber band, right? So pulling on this single rubber band and the 2 bands to the right as before would lead to the same total frictional drag as in **Figure 28**, right?

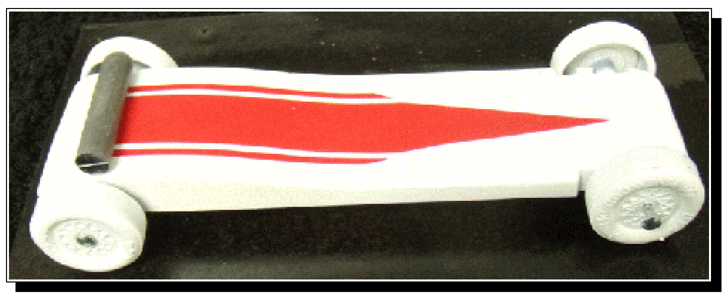


Figure 30 *- Pinewood Derby car, with an extra lead weight equally supported by rear wheels.*

Next **Figure 30** shows a PWD car with a lead weight centered and pushing down equally on the 2 rear wheels. Is the total frictional drag going to change if we take half the weight from the left rear and put it on the right rear as in **Fig 31**? **question 21**: **a** Yes **b** No

Figure 31 *- Pinewood Derby car, lead weight distribution moved closer to right rear wheel.*

In **Figure 28**, would it take the same net frictional force to slide all 208 cards separately? **question 22** : **a** Yes **b** No

In **Figure 28** or **29**, would it change net frictional drag if decks were turned to reduce contact area? **question 23** : **a** Yes **b** No

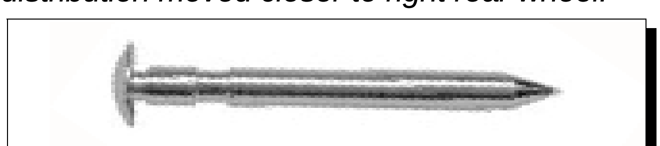


Figure 32 *- Grooved axle*

In **Figure 30**, would it change net frictional drag on the car if axles were grooved, like **Fig 32**, so less axle surface area was contacting the wheel bore surface? **question 24** : **a** Yes **b** No